Chapter 2: Mechanical Properties of Fluids

EXERCISES [PAGES 54 - 55]

Exercises | Q 1.1 | Page 54

Multiple Choice Question.
A hydraulic lift is designed to lift heavy objects of maximum mass 2000 kg. The area of cross-section of piston carrying the load is 2.25×10^{-2} m^2. What is the maximum pressure the smaller piston would have to bear?

1. **0.8711×10^6 N/m^2**
2. 0.5862×10^7 N/m^2
3. 0.4869×10^5 N/m^2
4. 0.3271×10^4 N/m^2

SOLUTION

0.8711×10^6 N/m^2

Exercises | Q 1.2 | Page 54

Multiple Choice Question.
Two capillary tubes of radii 0.3 cm and 0.6 cm are dipped in the same liquid. The ratio of heights through which the liquid will rise in the tubes is

1. 1: 2
2. **2: 1**
3. 1: 4
4. 4: 1

SOLUTION

2: 1

Explanation:

By using rise in capillary tube formula

$$h = \frac{2\,T\cos\theta}{\rho g r}$$

$$hA = \frac{2T(1)}{\rho(0.3)g} \quad ....(1)$$

$$hB = \frac{2T(1)}{\rho(0.4)g} \quad ....(2)$$

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Dividing (1) and (2) we get

$$\frac{hB}{hA} = \frac{0.2}{0.4} = 2$$

Hence, the ratio of heights in A and B is 2:1

Exercises | Q 1.3 | Page 54

Multiple Choice Question.
The energy stored in a soap bubble of diameter 6 cm and T = 0.04 N/m is nearly

1. **0.9×10^{-3} J**
2. 0.4×10^{-3} J
3. 0.7×10^{-3} J
4. 0.5×10^{-3} J

SOLUTION

0.9×10^{-3} J

Exercises | Q 1.4 | Page 54

Multiple Choice Question.
Two hailstones with radii in the ratio of 1: 4 fall from a great height through the atmosphere. Then the ratio of their terminal velocities is

1. 1: 2
2. 1: 12
3. **1: 16**
4. 1: 8

SOLUTION

1: 16

Exercises | Q 1.5 | Page 54

Multiple Choice Question.
Bernoulli theorem is based on the conservation of

1. linear momentum
2. mass
3. **energy**
4. angular momentum

SOLUTION

energy

Explanation:

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The principle behind the Bernoulli theorem is the law of conservation of energy. It states that energy can be neither created nor destroyed; it merely changes from one form to another.

Exercises | Q 2.1 | Page 54
Why is the surface tension of paints and lubricating oils kept low?

SOLUTION

For better wettability (surface coverage), the surface tension and angle of contact of paints and lubricating oils must be low.

Exercises | Q 2.2 | Page 54
How much amount of work is done in forming a soap bubble of radius r?

SOLUTION

We know that a bubble has two surfaces in contact with air, so the total surface area of the bubble will be

$= 2 \times (4 \pi R^2)$

$= 8 \pi R^2$

Now, work done=Surface tension × Increase in surface area

$= T \times (8 \pi R^2 - 0)$

$= 8\pi R^2 T$

Exercises | Q 2.3 | Page 54
What is the basis of Bernoulli's principle?

SOLUTION

Conservation of energy.

Exercises | Q 2.4 | Page 54
Why is a low-density liquid used as a manometric liquid in a physics laboratory?

SOLUTION

An open tube manometer measures the gauge pressure, $p - p_0 = h\rho g$, where p is the pressure being measured, p_0 is the atmospheric pressure, h is the difference in height between the manometric liquid of density ρ in the two arms. For a given pressure p, the product hρ is constant. That is, ρ should be small for h to be large. Therefore, for noticeably large h, a laboratory manometer uses a low-density liquid.

Exercises | Q 2.5 | Page 54
What is an incompressible fluid?

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SOLUTION

An incompressible fluid is one which does not undergo a change in volume for a large range of pressures. Thus, its density has a constant value throughout the fluid. In most cases, all liquids are incompressible.

Exercises | Q 3 | Page 54

Why two or more mercury drops form a single drop when brought in contact with each other?

SOLUTION

A spherical shape has the minimum surface area to volume ratio of all geometric forms. When two drops of a liquid are brought in contact, the cohe-sive forces between their molecules coalesce the drops into a single larger drop.

This is because, the volume of the liquid remaining the same, the surface area of the resulting single drop is less than the combined surface area of the smaller drops. The resulting decrease in surface energy is released into the environment as heat.

Proof: Let n droplets each of radius r coalesce to form a single drop of radius R. As the volume of the liquid remains constant,

volume of the drop = volume of n droplets

$$\therefore \frac{4}{5}\pi R^3 = n \times \frac{4}{3}\pi r^3$$

$$\therefore R^3 = nr^3$$

$$\therefore R = \sqrt[3]{n}r$$

Surface area of n droplets = $n \times 4\pi r^2$

Surface area of the drop = $4\pi R^2 = n^{2/3} \times 4\pi r^2$

$\therefore$ The change in the surface area

= surface area of drop - surface area of n droplets

$= 4\pi R^2 = (n^{2/3} - n)$

Since the bracketed term is negative, there is a decrease in surface area and a decrease in surface energy.

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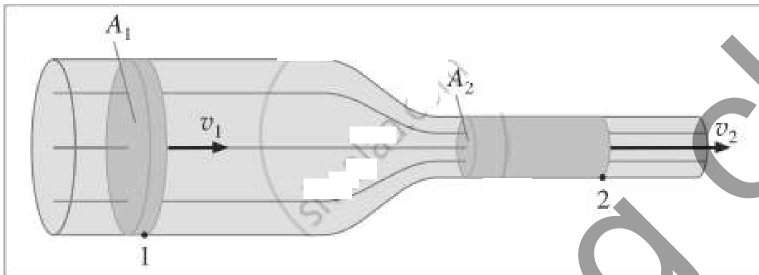
Exercises | Q 4 | Page 54

Why does velocity increase when water flowing in broader pipe enters a narrow pipe?

SOLUTION

When a tube narrows, the same volume occupies a greater length, as schematically shown in Figure. A_1 is the cross-section of the broader pipe and that of the narrower pipe is A_2.

By the equation of continuity, $v_2 = (A_1/A_2)v_1$



peed of fluid increases as it enters a narrower pipe (Not drawn to scale)

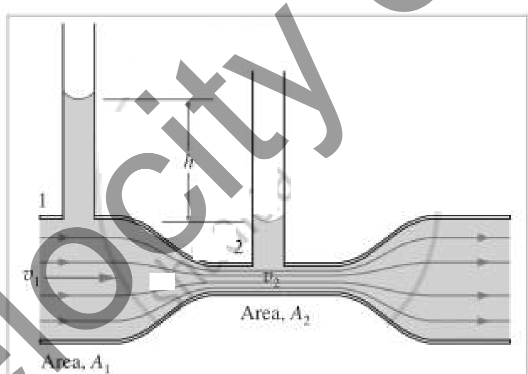
Since $A_1 /A_2 > v_2 > v_1$. For the same volume to pass points 1 and 2 in a given time, the speed must be greater at point 2.
The process is exactly reversible. If the fluid flows in the opposite direction, its speed decreases when the tube widens.

Exercises | Q 5 | Page 54

Why does the speed of a liquid increase and its pressure decrease when a liquid passes through a constriction in a horizontal pipe?

SOLUTION

Consider a horizontal constricted tube.



Let A_1 and A_2 be the cross-sectional areas at points 1 and 2, respectively. Let v_1 and v_2 be the corresponding flow speeds. ρ is the density of the fluid in the pipeline. By the equation of continuity,

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$v_1A_1 = v_2A_2$ ....(1)

$$\therefore \frac{v_2}{v_1} = \frac{A_1}{A_2} > 1 \quad (\because A_1 > A_2)$$

Therefore, the speed of the liquid increases as it passes through the constriction. Since the meter is assumed to be horizontal, from Bernoulli's equation we get,

$$p_1 = \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho_2^2$$

$$\therefore p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_1^2\left(\frac{A_1}{A_2}\right)^2 \quad \text{.....[from eq.(1)]}$$

$$\therefore p_1 - p_2 = \frac{1}{2}p v_1^2\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right] \quad \text{...(2)}$$

Again, since $A_1 > A_2$, the bracketed term is positive so that $p_1 > p_2$. Thus, as the fluid passes through the constriction or throat, the higher speed results in lower pressure at the throat.

Exercises | Q 6 | Page 54

Derive an expression for excess pressure inside a drop of liquid.

SOLUTION

Consider a liquid drop of radius R and surface tension T.

Due to surface tension, the molecules on the surface film experience the net force in the inward direction normal to the surface.
Therefore there is more pressure inside than outside.

Let p_1 be the pressure inside the liquid drop and p_0 be the pressure outside the drop.

Therefore, excess of pressure inside the liquid drop is,

$p = p_1 - p_0$

Due to excess pressure inside the liquid drop the free surface of the drop will experience the net force in outward direction due to which the drop will expand.

Let the free surface displace by dR under isothermal conditions.

Therefore, excess of pressure does the work in displacing the surface and that work will be stored in the form of potential energy.

The work done by an excess of pressure in displacing the surface is,

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dW = Force x displacement

= (Excess of pressure x surface area) x displacement of the surface

= $p \times 4\pi R^2 \times dR$ ....(1)

Increase in the potential energy is,

dU = surface tension x increases in area of the free surface

= $T[4\pi(R + dR)^2 - 4\pi R^2]$

= $T[4\pi(2RdR)]$ ....(2)

From (1) and (2)

$p \times 4\pi R^2 \times dR = T[4\pi(2RdR)]$

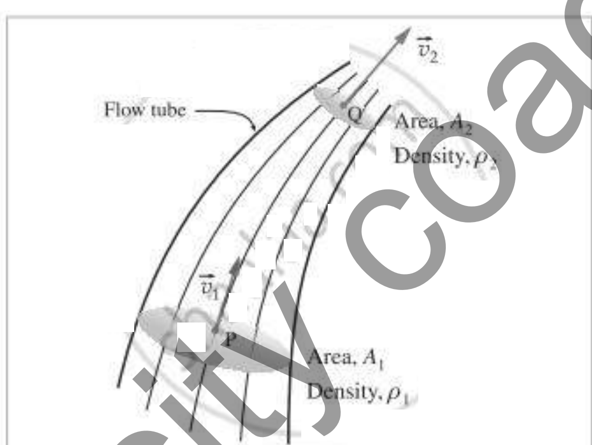
$$\Rightarrow p = \frac{2T}{R}$$

The above expression gives us the pressure inside a liquid drop.

Exercises | Q 7 | Page 54

Obtain an expression for conservation of mass starting from the equation of continuity.

SOLUTION



Consider a fluid in steady or streamline flow, that is its density is constant. The velocity of the fluid within a flow tube, while everywhere parallel to the tube, may change its magnitude. Suppose the velocity is $\vec{v}_1$, at point P and $\vec{v}_2$ at point Q. If A_1 and A_2 are the cross-sectional areas of the tube at these two points, the volume flux across A_1,

$$\frac{d}{dt}(V_1) = A_1 v_1$$

and that across A_2, $\frac{d}{dt}(V_2) = A_2 v_2$

By the equation of continuity of flow for a fluid,

$$A_1 v_1 = A_2 v_2$$

i.e. $\frac{d}{dt}(V_1) = \frac{d}{dt}(V_2)$

If ρ_1 and ρ_2 are the densities of the fluid at P and Q, respectively, the mass flux across A_1, $\frac{d}{dt}(m_1)$

$$= \frac{d}{dt}(\rho_1 V_1) = A_1 \rho_1 v_1$$

and that across A_2, $\frac{d}{dt}(m_2) = \frac{d}{dt}(\rho_2 V_2) = A_2 \rho_2 v_2$

Since no fluid can enter or leave through the boundary of the tube, the conservation of mass requires the mass fluxes to be equal, i.e.,

$$\frac{d}{dt}(m_1) = \frac{d}{dt}(m_2)$$

i.e. $A_1\rho_1 v_1 = A_2 \rho_2 v_2$

i.e. $A\rho v$ = constant which is the required expression.

Exercises | Q 8 | Page 54

Explain the capillary action.

SOLUTION

When a capillary tube is partially immersed in a wetting liquid, there is a capillary rise and the liquid meniscus inside the tube is concave, as shown in Figure (a).

Consider four points A, B, C, D, of which point A is just above the concave meniscus inside the capillary, and point B is just below it. Points C and D are just above and below the free liquid surface outside.

Let P_A, P_B, P_C, and P_D be the pressures at points A, B, C, and D respectively.

Now, $P_A = P_C$ = atmospheric pressure

The pressure is the same on both sides of the free surface of a liquid, so that

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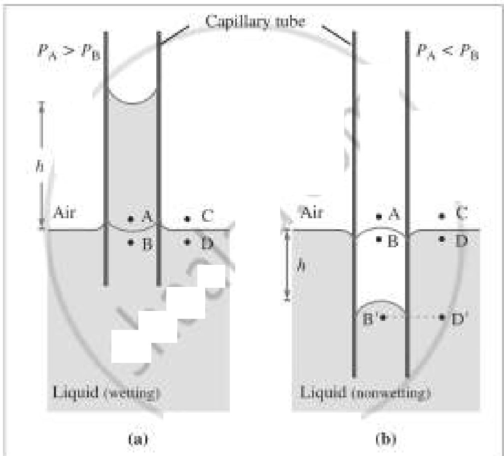

Explanation of (a) capillary rise
(b) capillary depression

$P_C = P_D$

$\therefore P_A = P_D$

The pressure on the concave side of a meniscus is always greater than that on the convex side, so that

$P_A > P_B$

$\therefore P_D > P_B$ ....($\because$ $P_A = P_D$)

The excess pressure outside presses the liquid up the capillary until the pressures at B and D (at the same horizontal level) equalize, i.e., P_B becomes equal to P_D. Thus, there is a capillary rise.

For a non-wetting liquid, there is capillary depression and the liquid meniscus in the capillary tube is convex, as shown in Figure (b).

Consider again four points A, B, C, and D when the meniscus in the capillary tube is at the same level as the free surface of the liquid. Points A and B are just above and below the convex meniscus. Points C and D are just above and below the free liquid surface outside.

The pressure at B(P_B) is greater than that at A(P_A). The pressure at A is the atmospheric pressure H and at D, $P_D \simeq H = P_A$. Hence, the hydrostatic pressure at the same levels at B and D are not equal, $P_B > P_D$. Hence, the liquid flows from B to D, and the level of the liquid in the capillary falls. This continues till the pressure at B' is the same as that D', that is till the pressures at the same level are equal.

Exercises | Q 9 | Page 54
Derive an expression for capillary rise for a liquid having a concave meniscus.

SOLUTION

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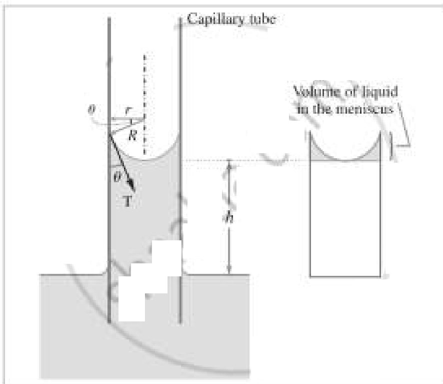
Consider a capillary tube of radius r partially immersed into a wetting liquid of density ρ. Let the capillary rise be h and θ be the angle of contact at the edge of contact of the concave meniscus and glass figure. If R is the radius of curvature of the meniscus then from the figure, r = R cos θ.



Analysing capillary action using
Laplace's law for a spherical membrane

Surface tension T is the tangential force per unit length acting along the contact line. It is directed into the liquid making an angle θ with the capillary wall. We ignore the small volume of the liquid in the meniscus. The gauge pressure within the liquid at a depth h, i.e., at the level of the free liquid surface open to the atmosphere, is

$p - p_0 = \rho g h$ ...(1)

By Laplace's law for a spherical membrane, this gauge pressure is

$$p - p_0 = \frac{2T}{R} \quad ...(2)$$

$$\therefore h\rho g = \frac{2T}{R} = \frac{2T\cos\theta}{r}$$

$$\therefore h = \frac{2T\cos\theta}{r\rho g} \quad .....(3)$$

Thus, the narrower the capillary tube, the greater is the capillary rise.

From Eq. (3),

$$T = \frac{h\rho r g}{2T\cos\theta} \quad ....(4)$$

Equations (3) and (4) are also valid for capillary depression h of a non-wetting liquid. In this case, the meniscus is convex and θ is obtuse. Then, cos θ is negative but so is h, indicating a fall or depression of the liquid in the capillary. T is positive in both cases.

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[**Note:** The capillary rise h is called Jurin height, after James Jurin who studied the effect in 1718. For capillary rise, Eq. (3) is also called the ascent formula.]

Exercises | Q 10 | Page 55

Find the pressure 200 m below the surface of the ocean if pressure on the free surface of liquid is one atmosphere. (Density of seawater = 1060 kg/m^3)

SOLUTION

Data: h = 200 m, p = 1060 kg/m^3,

$P_0 = 1.013 \times 10^5$ Pa, g = 9.8 m/s^2

Absolute pressure,

$P = P_0 + hpg$

$= (1.013 \times 10^3) + (200 \times 1060) \times (9.8)$

$= (1.013 \times 10^5) + (20.776 \times 10^5)$

$= 21.789 \times 10^5$

= 2.1789 MPa

Exercises | Q 11 | Page 55

In a hydraulic lift, the input piston had surface area 30 cm^2 and the output piston has surface area of 1500 cm^2. If a force of 25 N is applied to the input piston, calculate weight on output piston.

SOLUTION

Data: $A_1 = 30\ cm^2 = 3 \times 10^{-3}\ m^2$

$A_2 = 1500\ cm^2 = 0.15\ m^2$, $F_1 = 25$ N

By Pascal's law,

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

∴ The force on the output piston,

$$F_2 = F_1 \frac{A_2}{A_1} = (25)\frac{0.15}{3 \times 10^{-3}}$$

$= 25 \times 50$

= 1250 N

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Exercises | Q 12 | Page 55

Calculate the viscous force acting on a raindrop of diameter 1 mm, falling with a uniform velocity of 2 m/s through air. The coefficient of viscosity of air is 1.8×10^{-5} N.s/m^2.

SOLUTION

Data: d = 1 mm, v_0 = 2 m/s, $\eta = 1.8 \times 10^{-5}$ N.s/m^2

$r = \frac{d}{2} = 0.5$ mm $= 5 \times 10^{-4}$ m

By Stokes' law, the viscous force on the raindrop is

$f = 6\pi\eta r v_0$

$= 6 \times 3.142\ (1.8 \times 10^{-5}\ \text{N.s/m}^2 \times 5 \times 10^{-4}\ \text{m})(2\ \text{m/s})$

$= 3.394 \times 10^{-7}$ N

Exercises | Q 13 | Page 55

A horizontal force of 1 N is required to move a metal plate of area 10^{-2} m^2 with a velocity of 2×10^{-2} m/s, when it rests on a layer of oil 1.5×10^{-3} m thick. Find the coefficient of viscosity of oil.

SOLUTION

Data: F = 1 N, A = 10^{-2} m^2, $v_0 = 2 \times 10^{-2}$ m/s,

$y = 1.5 \times 10^{-3}$ m

Velocity gradient, $\frac{dv}{dy} = \frac{2 \times 10^{-2}}{1.5 \times 10^{-3}} = \frac{40}{3}\, s^{-1}$

Viscous force, $F = \eta \frac{dv}{dy}$

∴ The coefficient of viscosity is

$\eta = \frac{F}{A(dv/dy)}$

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$$= \frac{1\text{N}}{(10^{-2}\text{m}^2)(40/3\text{s}^{-1})} = \frac{30}{4} = 7.5 \text{ Pa.s}$$

Exercises | Q 14 | Page 55

With what terminal velocity will an air bubble 0.4 mm in diameter rise in a liquid of viscosity 0.1 Ns/m^2 and specific gravity 0.9? Density of air is 1.29 kg/m^3.

SOLUTION

Data: d = 0.4 mm, η = 0.1 Pa.s, ρ_L = 0.9 × 10^3 kg/m^3 = 900 kg/m^3, ρ_{air} = 1.29 kg/m^3, g = 9.8 m/s^2.

Since the density of air is less than that of oil, the air bubble will rise up through the liquid. Hence, the viscous force is downward. At terminal velocity, this downward viscous force is equal in magnitude to the net upward force.

Viscous force = buoyant force - the gravitational force

$$\therefore 6\pi\eta r v_t = \frac{4}{3}\pi r^3(\rho_L - \rho_{air})g$$

∴ The terminal velocity,

$$v_t = \frac{2r^2 g(\rho_L - \rho)}{9\eta}$$

$$= \frac{2(2 \times 10^{-4}\text{m})^2(9.8\text{ m/s}^2)(900 - 1.29)}{9(0.1\text{ Pa.s})}$$

$$= \frac{2 \times 4 \times 9.8 \times 898.7 \times 10^{-8}}{0.9}$$

$$= 7.829 \times 10^4 \times 10^{-8}$$

$= 7.829 \times 10^{-4}$ m/s = 0.7829 mm/s ......[The negative sign indicates that the bubble rises up]

Exercises | Q 15 | Page 55

The speed of water is 2m/s through a pipe of internal diameter 10 cm. What should be the internal diameter of the nozzle of the pipe if the speed of the water at nozzle is 4 m/s?

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SOLUTION

Data: d_1 = 10 cm = 0.1 m, v_1 = 2 m/s, v_2 = 4 m/s

By the equation of continuity, the ratio of the speed is

$$\frac{v_1}{v_2} = \frac{A_2}{A_1} = \left(\frac{d_2}{d_1}\right)^2$$

$$\therefore \frac{d_2}{d_1} = \sqrt{\frac{v_1}{v_2}} = \sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}} = 0.707$$

$\therefore d_2 = 0.707\, d_1 = 0.707(0.1 \text{ m}) = 0.0707$ m

Exercises | Q 16 | Page 55

With what velocity does water flow out of an orifice in a tank with gauge pressure 4 × 10^5 N/m^2 before the flow starts? Density of water = 1000 kg/m^3.

SOLUTION

Data: $p - p_0 = 4 \times 10^5$ Pa, $\rho = 10^3$ kg/m^3

If the orifice is at a depth h from the water surface in a tank, the gauge pressure there is

$p - p_0 = h\rho g$ ....(1)

By Toricelli's law of efflux, the velocity of efflux,

$$v = \sqrt{2gh} \quad ...(2)$$

Substituting for h from Eq.(1),

$$v = \sqrt{2g\frac{p - p_0}{\rho g}} = \sqrt{\frac{(2p - p_0)}{\rho}}$$

$$= \sqrt{\frac{2(4 \times 10^5)}{10^3}} = 20\sqrt{2} = 28.28 \text{ m/s}.$$

Exercises | Q 17 | Page 55

The pressure of water inside a closed pipe is 3 × 10^5 N/m^2. This pressure reduces to 2 × 10^5 N/m^2 on opening the valve of the pipe. Calculate the speed of water flowing through the pipe. [Density of water = 1000 kg/m^3].

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SOLUTION

Data: $p_1 = 3 \times 10^5$ Pa, $v_1 = 0$, $p_2 = 2 \times 10^5$ Pa,
$\rho = 10^3$ kg/m^3

Assuming the potential head to be zero, i.e., the pipe to be horizontal, the Bernoulli equation is

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

$$\therefore v_2^2 = \frac{2(p_1 - p_2)}{\rho} \quad ....[\because v_1 = 0]$$

$$= \frac{2(3-2)\times 10^5}{10^3} = 200$$

$$\therefore v_2 = 10\sqrt{2} = 14.14 \text{m/s}$$

Exercises | Q 18 | Page 55

Calculate the rise of water inside a clean glass capillary tube of radius 0.1 mm, when immersed in water of surface tension 7×10^{-2} N/m. The angle of contact between water and glass is zero, the density of water = 1000 kg/m^3, g = 9.8 m/s^2.

SOLUTION

Given :

- Radius of capillary tube = 0.1 mm
- Surface tension of water = 7×10^{-2} N/m
- Angle of contact = 0°
- Density of water = 1000 kg/m
- Acceleration due to gravity = 9.8 m/s

To find:

- Height of water column inside the capillary tube.

Formula:

When a capillary tube of radius 'r' is dipped in a liquid of density ρ and surface tension T, the liquid rises or falls through a distance,

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$$H = \frac{2T\cos\theta}{\rho g r}$$

$$H = \frac{2 \times 7 \times 10^{-2} \times \cos\theta}{1000 \times 9.8 \times 0.1 \times 10^{-3}}$$

H = 0.142 m

Exercises | Q 19 | Page 55

An air bubble of radius 0.2 mm is situated just below the water surface. Calculate the gauge pressure. Surface tension of water = 7.2 × 10^{-2} N/m.

SOLUTION

Data: R = 2 × 10^{-4} m, T = 7.2 × 10^{-2} N/m,

p = 10^3 kg/m^3

The gauge pressure inside the bubble = $\frac{2T}{R}$

$$= \frac{2\left(7.2 \times 10^{-2}\right)}{2 \times 10^{-4}}$$

= 7.2 × 10^2

= 720 Pa

Exercises | Q 20 | Page 55

Twenty-seven droplets of water, each of radius 0.1 mm coalesce into a single drop. Find the change in surface energy. Surface tension of water is 0.072 N/m.

SOLUTION

r = 0.1 mm = 0.1 × 10^{-3} m, T = 0.072 N/m

Let R be the radius of the single drop formed due to the coalescence of 27 droplets of mercury. Volume of 27 droplets = volume of the single drop as the volume of the liquid remains constant.

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$$\therefore 27 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

$$\therefore 27r^3 = R^3$$

$\therefore$ 3r = R

Surface area of 27 droplets = $27 \times 4\pi r^2$ Surface area of single drop = $4\pi R^2$

$\therefore$ Decrease in surface area = $27 \times 4\pi r^2 - 4\pi R^2$

$$= 4\pi\left(27r^2 - R^2\right)$$

$$= 4\pi\left[27r^2 - (3r)^2\right]$$

$$= 4\pi \times 18r^2$$

$\therefore$ The energy released

= surface tension × decrease in surface area

= T × 4π × $18r^2$

= 0.072 × 4 × 3.142 × 18 × $(1 \times 10^{-4})^2$

= 1.628×10^{-7} J.

Exercises | Q 21 | Page 55

A drop of mercury of radius 0.2 cm is broken into 8 droplets of the same size. Find the work done if the surface tension of mercury is 435.5 dyn/cm.

SOLUTION

Let R be the radius of the drop and r be the radius of each droplet.

Data: R = 0.2 cm, n = 8, T = 435.5 dyn/cm

As the volume of the liquid remains constant, volume of n droplets = volume of the drop

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$$\therefore n \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

$$\therefore r^3 = \frac{R^3}{n}$$

$$\therefore r = \frac{R}{\sqrt[3]{n}} = \frac{R}{\sqrt[3]{8}} = \frac{R}{2}$$

Surface area of the drop = $4\pi R^2$

Surface area of n droplets = $n \times 4\pi R^2$

$\therefore$ The increase in the surface area

= surface area of n droplets – surface area of drop

$= 4\pi(nr^2 - R^2) = 4\pi\left(8 \times \frac{R^2}{4} - R^2\right)$

$= 4\pi(2 - 1)R^2 = 4\pi R^2$

$\therefore$ The work done

= surface tension × increase in surface area

$= T \times 4\pi R^2 = 4.5.5 \times 4 \times 3.142 \times (0.2)^2$

$= 2.19 \times 10^2$ ergs $= 2.19 \times 10^{-5}$ J

Exercises | Q 22 | Page 55

How much work is required to form a bubble of 2 cm radius from the soap solution having surface tension 0.07 N/m?

SOLUTION

$r = 2$ cm $= 2 \times 10^{-2}$ m, $T = 0.07$ N/m

Initial surface area of soap bubble = 0

Final surface area of soap bubble = $2 \times 4\pi r^2$

$\therefore$ Increase in surface area = $2 \times 4\pi r^2$

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The work done

= surface tension × increase in surface area

$= T \times 2 \times 4\pi r^2$

The work done $= 0.07 \times 8 \times 3.142 \times (2 \times 10^{-2})^2$

$= 7.038 \times 10^{-4}$ J.

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A rectangular wire frame of size 2 cm × 2 cm is dipped in a soap solution and taken out. A soap film is formed If the size of the film is a change to 3 cm × 3 cm, calculate the work done in the process. [Surface tension of the soap film is 3×10^{-2} N/m]

SOLUTION

Data: $A_1 = 2 \times 2\ cm^2 = 4 \times 10^{-4}\ m^2$,

$A_2 = 3 \times 3\ cm^2 = 9 \times 10^{-4}\ m^2$, $T = 3 \times 10^{-2}$ N/m

As the film has two surfaces, the work done is

$W = 2T(A_2 - A_1)$

$= 2(3 \times 10^{-2})(9 \times 10^{-4} - 4 \times 10^{-4})$

$= 3.0 \times 10^{-5}$ J = 30 μ J.

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