

Line and Plane

EXERCISE 6.1 [PAGES 200 - 201]

Exercise 6.1 | Q 1 | Page 200

Find the vector equation of the line passing through the point having position vector $-2\hat{i} + \hat{j} + \hat{k}$ and parallel to vector $4\hat{i} - \hat{j} + 2\hat{k}$.

Solution:

The vector equation of the line passing through A(\bar{a}) and parallel to the vector \bar{b} is $\bar{r} = \bar{a} + \lambda \bar{b}$, where λ is a scalar.

:. the vector equation of the line passing through the point having position vector $-2\hat{i} + \hat{j} + \hat{k} \text{ and parallel to the vector } 4\hat{i} - \hat{j} + 2\hat{k} \text{ is}$ $\bar{r} = \left(-2\hat{i} + \hat{j} + \hat{k}\right) + \lambda \left(4\hat{i} - \hat{j} + 2\hat{k}\right).$

Exercise 6.1 | Q 2 | Page 200

Find the vector equation of the line passing through points having position vector $3\hat{i} + 4\hat{j} - 7\hat{k}$ and $6\hat{i} - \hat{j} + \hat{k}$.

Solution:

The vector equation of the line passing through the $A(\bar{a})$ and $B(\bar{b})$ is $\bar{r} = \bar{a} + \lambda (\bar{b} - \bar{a})$, λ is a scalar.

 \therefore the vector equation of the line passing through the points having position vector

$$\begin{split} &3\hat{i}+4\hat{j}-7\hat{k} \ \text{and} \ 6\hat{i}-\hat{j}+\hat{k} \text{ is} \\ &\overline{r}=\left(3\hat{i}+4\hat{j}-7\hat{k}\right)+\lambda\Big[\left(6\hat{i}-\hat{j}+\hat{k}\right)-\left(3\hat{i}+4\hat{j}-7\hat{k}\right)\Big] \\ &\text{i.e. } \overline{r}=\left(3\hat{i}+4\hat{j}-7\hat{k}\right)+\lambda\Big(3\hat{i}-5\hat{j}+8\hat{k}\Big). \end{split}$$

Exercise 6.1 | Q 3 | Page 200

Find the vector equation of line passing through the point having position vector $5\hat{i} + 4\hat{j} + 3\hat{k}$ and having direction ratios –3, 4, 2.



Solution:

Let A be the point whose position vector is a = $5\hat{i} + 4\hat{j} + 3\hat{k}$.

Let $\overline{\mathbf{b}}$ be the vector parallel to the line having direction ratio = -3, 4, 2

Then, $ar{b}=-3\hat{i}+4\hat{j}+2\hat{k}$

The vector equation of the line passing through $A(\bar{a})$ and parallel to \bar{b} is $\bar{r} = \bar{a} + \lambda \bar{b}$, where λ is a scalar.

 \div the required vector equation of the line is

$$ar{\mathbf{r}}=5\,\hat{\mathbf{i}}+4\,\hat{\mathbf{j}}+3\hat{\mathbf{k}}+\lambda\Big(-3\,\hat{\mathbf{i}}+4\,\hat{\mathbf{j}}+2\hat{\mathbf{k}}\Big).$$

Exercise 6.1 | Q 4 | Page 200

Find the vector equation of the line passing through the point having position vector $\hat{i} + 2\hat{j} + 3\hat{k}$ and perpendicular to vectors $\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} - \hat{j} + \hat{k}$.

Solution:

Let $\bar{\mathbf{b}}=\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}} ~\mathrm{and}~\bar{\mathbf{c}}=2\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{\mathbf{k}}$

The vector perpendicular to the vectors $\bar{\mathbf{b}}$ and $\bar{\mathbf{c}}$ is given by

$$\bar{\mathbf{b}} \times \bar{\mathbf{c}} = \begin{vmatrix} \hat{\mathbf{i}} \\ 1 & 1 & 1 \\ 2 & -1 & 1 \end{vmatrix}$$

= $\hat{\mathbf{i}}(1+1) - \hat{\mathbf{j}}(1-2) + \hat{\mathbf{k}}(-1-2)$
= $2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$

Since the line is perpendicular to the vector $\bar{\mathbf{b}}$ and $\bar{\mathbf{c}}$, it is parallel to $\bar{\mathbf{b}} \times \bar{\mathbf{c}}$.

The vector equation of the line passing through

 $A(\bar{a})$ and parallel to $\bar{b} \times \bar{c}$ is $\bar{r} = \bar{a} + \lambda (\bar{b} \times \bar{c})$, where λ is a scalar.

Here,
$$ar{\mathbf{a}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$$

Hence, the vector equation of the required line is $\bar{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda \left(2\hat{i} + \hat{j} - 3\hat{k}\right)$.

Exercise 6.1 | Q 5 | Page 200



Find the vector equation of the line passing through the point having position vector $-\hat{i} - \hat{j} + 2\hat{k}$ and parallel to the line $\bar{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(3\hat{i} + 2\hat{j} + \hat{k})$.

Solution:

Let A be point having position vector $\bar{a}=-\,\hat{i}\,-\,\hat{j}\,+\,2\hat{k}$ The required line is parallel to the line

$$\overline{\mathbf{r}} = \left(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}} + \lambda\left(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}\right)\right)$$

∴ it is parallel to the vector

$$\bar{b}=3\hat{i}+2\hat{j}+\hat{k}$$

The vector equation of the line passing through $A(\bar{a})$ and parallel to \bar{b} is $\bar{r} = \bar{a} + \lambda \bar{b}$ where λ is a scalar.

... the required vector equation of the line is

$$ar{\mathbf{r}} = \left(-\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}
ight) + \lambda \left(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}
ight).$$

Exercise 6.1 | Q 6 | Page 200

Find the Cartesian equations of the line passing through A(-1, 2, 1) and having direction ratios 2, 3, 1.

Solution: The cartesian equations of the line passing through (x_1, y_1, z_1) and having direction ratiosa,b,c are and having direction ratios a,b,c are

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

 \therefore the cartesian equations of the line passing through the point (-1, 2, 1) and having direction ratios 2, 3, 1 are

$$\frac{x - (-1)}{2} = \frac{y - 2}{3} = \frac{z - 1}{1}$$

i.e. $\frac{x + 1}{2} - \frac{y - 2}{3} = \frac{z - 1}{1}$

Exercise 6.1 | Q 7 | Page 200



Find the Cartesian equations of the line passing through A(2, 2, 1) and B(1, 3, 0).

Solution: The cartesian equations of the line passing through the points (x_1, y_1, z_1) and

(x₂, y₂, z₂) are

 $\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$ Here, $(x_1, y_1, z_1) \equiv (2, 2, 1)$ and $(x_2, y_2, z_2) \equiv (1, 3, 0)$

 \therefore the required cartesian equations are

$$\frac{x-2}{1-2} = \frac{y-2}{3-2} = \frac{z-1}{0-1}$$

i.e. $\frac{x-2}{-1} = \frac{y-2}{1} = \frac{z-1}{-1}$

Exercise 6.1 | Q 8 | Page 200

A(-2, 3, 4), B(1, 1, 2) and C(4, -1, 0) are three points. Find the Cartesian equations of the line AB and show that points A, B, C are collinear.

Solution: We find the cartesian equations of the line AB. The cartesian equations of the line passing through the points (x_1, y_1, z_1) and (x_2, y_2, z_2) are

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

Here, (x₁, y₁, z₁) = (-2, 3, 4) and (x₂, y₂, z₂) = (4, -1, 0)

: the required cartesian equations of the line AB are

$$\frac{x - (-2)}{4 - (-2)} = \frac{y - 3}{-1 - 3} = \frac{z - 4}{0 - 4}$$

$$\therefore \frac{x + 2}{6} = \frac{y - 3}{-4} = \frac{z - 4}{-4}$$

$$\therefore \frac{x + 2}{3} = \frac{y - 3}{-2} = \frac{z - 4}{-2}$$

$$C = (4, -1, 0)$$

For x = 4, $\frac{x + 2}{3} = \frac{4 + 2}{3} = 2$



For y = -1,
$$\frac{y-3}{-2} = \frac{-1-3}{-2} = 2$$

For z = 0, $\frac{z-4}{-2} = \frac{0-4}{-2} = 2$

: coordinates of C satisfy the equations of the line AB.

: C lies on the line passing through A and B.

Hence, A, B, C are collinear.

Exercise 6.1 | Q 9 | Page 200

Show that the lines given by

$$\frac{x+1}{-10} = \frac{y+3}{-1} = \frac{z-4}{1}$$
 and $\frac{x+10}{-1} = \frac{y+1}{-3} = \frac{z-1}{4}$

intersect. Also, find the coordinates of their point of intersection.

Solution: The equations of the lines are

$$\frac{x+1}{-10} = \frac{y+3}{-1} = \frac{z-4}{1} = \lambda \qquad \dots (say)\dots(1)$$

and
$$\frac{x+10}{-1} = \frac{y+1}{-3} = \frac{z-1}{4} = \mu \qquad \dots (say)\dots(2)$$

From (1), $x=-1-10\lambda,\,y=-3-\lambda$, $z=4+\lambda$

: the coordinates of any point on the line (1) are $(-1 - 10\lambda, -3 - \lambda, 4 + \lambda)$

From (2), $x = -10 - \mu$, $y = -1 - 3\mu$, $z = 1 + 4\mu$ \therefore the coordinates of any point on the line (2) are $(-10 - \mu, -1 - 3\mu, 1 + 4\mu)$ Lines (1) and (2) intersect, if $(-1 - 10\lambda, -3 - \lambda, 4 + \lambda) = (-10 - \mu, -1 - 3\mu, 1 + 4\mu)$ \therefore the equation $-1 - 10\lambda = -10 - \mu, -3 - \lambda = -1 - 3\mu$ and $4 + \lambda = 1 + 4\mu$ are

simultaneously true.

Solving the first two equations, we get, $\lambda = 1$, and $\mu = 1$.

These values of λ and μ satisfy the third equation also.

 \therefore the lines intersect.

Putting $\lambda = 1$ in $(-1 - 10\lambda, -3 - \lambda, 4 + \lambda)$ or $\mu = 1$ in $(-10 - \mu, -1 - 3\mu, 1 + 4\mu)$, we get the point of intersection (-11, -4, 5).



Exercise 6.1 | Q 10 | Page 200

A line passes through (3, -1, 2) and is perpendicular to lines $\bar{\mathbf{r}} = (\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) + \lambda (2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}) \text{ and } \bar{\mathbf{r}} = (2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}) + \mu (\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$. Find its equation.

Solution:

The line $\bar{\mathbf{r}} = (\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) + \lambda (2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}})$ is parallel to the vector $\bar{\mathbf{b}} = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and the line $\bar{\mathbf{r}} = (2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}) + \mu (\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ is parallel to the vector. $\bar{\mathbf{c}} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$.

The vector perpendicular to the vectors $\bar{\mathbf{b}}~\mathbf{and}~\bar{\mathbf{c}}$ is given by

The vector perpendicular to the vectors $ar{\mathbf{b}}$ and $ar{\mathbf{c}}$ is given by

$$\bar{\mathbf{b}} \times \bar{\mathbf{c}} = \begin{vmatrix} \hat{\mathbf{i}} \\ 2 & -2 & 1 \\ 1 & -2 & 2 \end{vmatrix}$$

= $\hat{\mathbf{i}}(-4+2) - \hat{\mathbf{j}}(4-1) + \hat{\mathbf{k}}(-4+2)$
= $-2\hat{\mathbf{i}} - 3\hat{\mathbf{i}} - 2\hat{\mathbf{k}}$

Since the required line is perpendicular to the given lines,

it is perpendicular to both $\overline{\mathbf{b}}$ and $\overline{\mathbf{c}}$.

 \therefore It is parallel to $\bar{\mathbf{b}}\times\bar{\mathbf{c}}$

The equation of the line passing through $\mathbf{A}(\bar{a})$ and parallel to $\bar{\mathbf{b}}$ and $\bar{\mathbf{c}}$ is $\bar{\mathbf{r}} = \bar{\mathbf{a}} + \lambda (\bar{\mathbf{b}} \times \bar{\mathbf{c}})$, where λ is a scalar. Here, $\bar{\mathbf{a}} = 3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ \therefore the equation of the required line is $\bar{\mathbf{r}} = (3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) + \lambda (-2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$ or $\bar{\mathbf{r}} = (3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) + \mu (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$, where $\mu = -\lambda$.

Exercise 6.1 | Q 11 | Page 201



Show that the line
$$\frac{x-2}{1} = \frac{y-4}{2} = \frac{z+4}{-2}$$
 passes through the origin.

Solution:

The equation of the line is

$$\frac{x-2}{1} = \frac{y-4}{2} = \frac{z+4}{-2}$$

The coordinates of the origin O are (0, 0, 0)

For x = 0,
$$\frac{x-2}{1} = \frac{0-2}{1} = -2$$

For y = 0, $\frac{y-4}{2} = \frac{0-4}{2} = -2$
For z = 0, $\frac{z+4}{-2} = \frac{0+4}{-2} = -2$

 \therefore coordinates of the origin O satisfy the equation of the line.

Hence, the line passes through the origin.

EXERCISE 6.2 [PAGE 207]

Exercise 6.2 | Q 1 | Page 207

Find the length of the perpendicular (2, -3, 1) to the line $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z+1}{-1}$.

Solution1:

Let PM be the perpendicular drawn from the point P(2, -3, 1) to the line $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z+1}{-1} = \lambda$...(Say)

The coordinates of any point on the line are given by

 $\begin{aligned} x &= -1 + 2\lambda, \ y &= 3 + 3\lambda, \ z &= -1 - \lambda \\ \text{Let the coordinates of M be} \\ (-1 + 2\lambda, 3 + 3\lambda, -1 - \lambda) & \dots(1) \\ \text{The direction ratios of PM are} \\ -1 + 2\lambda - 2, \ 3 + 3\lambda + 3, -1 - \lambda - 1 \\ \text{i.e. } 2\lambda - 3, \ 3\lambda + 6, -\lambda - 2 \end{aligned}$



The direction ratios of the given line are 2, 3, -1. Since PM is perpendicular to the given line, we get $2(2\lambda - 3) + 3(3\lambda + 6) - 1(-\lambda - 2) = 0$ $\therefore 4\lambda - 6 + 9\lambda + 18 + \lambda + 2 = 0$ $\therefore 14\lambda + 14 = 0$ $\therefore \lambda = -1$. Put $\lambda = -1$ in (1), the coordinates of M are (-1 - 2, 3 - 3, -1 + 1) i.e. (-3, 0, 0). \therefore length of perpendicular from P to the given line = PM

$$= \sqrt{(-3-2)^2 + (0+3)^2 + (0-1)^2}$$

= $\sqrt{(25+9+1)}$
= $\sqrt{35}$ units.

Solution2:

We know that the perpendicular distance from the point

$$\begin{split} \mathbf{P} \left| \bar{a} \right| \text{ to the line } \mathbf{\bar{r}} &= \mathbf{\bar{a}} + \lambda \mathbf{\bar{b}} \text{ is given by} \\ \sqrt{\left| \left| \bar{a} - \mathbf{\bar{a}} \right|^2 - \left[\frac{\left(\bar{a} - \bar{a} \right) \cdot \mathbf{\bar{b}}}{\left| \mathbf{\bar{b}} \right|} \right]^2} \qquad \dots (1) \\ \text{Here, } \mathbf{\bar{a}} &= 2\mathbf{\hat{i}} - 3\mathbf{\hat{j}} + \mathbf{\hat{k}}, \mathbf{\bar{a}} = -\mathbf{\hat{i}} + 3\mathbf{\hat{j}} - \mathbf{\hat{k}}, \mathbf{\bar{b}} = 2\mathbf{\hat{i}} + 3\mathbf{\hat{j}} - \mathbf{\hat{k}} \\ \therefore \mathbf{\bar{a}} - \mathbf{\bar{a}} &= \left(2\mathbf{\hat{i}} - 3\mathbf{\hat{j}} + \mathbf{\hat{k}} \right) - \left(-\mathbf{\hat{i}} + 3\mathbf{\hat{j}} - \mathbf{\hat{k}} \right) \\ &= 3\mathbf{\hat{i}} - 6\mathbf{\hat{j}} + 2\mathbf{\hat{k}} \end{split}$$



$$\left\| \overline{\alpha} - \overline{a} \right\|^{2} = 3^{2} + (-6)^{2} + 2^{2} = 9 + 36 + 4 = 49$$
Also, $\left(\overline{\alpha} - \overline{a} \right) \cdot \overline{b} = \left(3\hat{i} - 6\hat{j} + 2\hat{k} \right) \cdot \left(2\hat{i} + 3\hat{j} - \hat{k} \right)$

$$= (3)(2) + (-6)(3) + (2)(-1)$$

$$= 6 - 18 - 2$$

$$= -14$$

$$\begin{split} \left| \bar{b} \right| &= \sqrt{2^2 + 3^2 + (-1)^2} = \sqrt{14} \\ \text{Substitutng these values in (1), we get} \\ \text{length of perpendicular from P to given line} \\ &= \mathsf{PM} \\ &= \sqrt{49 - \left(\frac{-14}{\sqrt{14}}\right)^2} \\ &= \sqrt{49 - 14} \\ &= \sqrt{35} \text{ units.} \end{split}$$

Exercise 6.2 | Q 2 | Page 207

Find the co-ordinates of the foot of the perpendicular drawn from the point $2\hat{i} - \hat{j} + 5\hat{k}$ to the line $\bar{r} = (11\hat{i} - 2\hat{j} - 8\hat{k}) + \lambda(10\hat{i} - 4\hat{j} - 11\hat{k})$. Also find the length of the perpendicular.

Solution:

Let M be the foot of perpendicular drawn from the point $P\Big(2\hat{i}-\hat{j}+5\hat{k}\Big)$ on the line

$$\overline{\mathbf{r}} = \Big(11\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 8\hat{\mathbf{k}}\Big) + \lambda\Big(10\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 11\hat{\mathbf{k}}\Big).$$

Let the position vector of the point M be

$$\Bigl(11\hat{\mathrm{i}}-2\hat{\mathrm{j}}-8\hat{k}\Bigr)+\lambda\Bigl(10\hat{\mathrm{i}}-4\hat{\mathrm{j}}-11\hat{k}\Bigr)$$



$$= (11 + 10\lambda)\hat{i} + (-2 - 4\lambda)\hat{j} + (-8 - 11\lambda)\hat{k}.$$

Then \overline{PM} = Position vector of M - Position vector of P

$$= \left[(11 + 10\lambda)\hat{i} + (-2 - 4\lambda)\hat{j} + (-8 - 11\lambda)\hat{k} \right] - \left(2\hat{i} - \hat{j} + 5\hat{k}\right)$$

$$= (9 + 10\lambda)\hat{i} + (-1 - 4\lambda)\hat{j} + (-13 - 11\lambda)\hat{k}$$
Since PM is perpendicular to the given line which is parallel to
 $\bar{b} = 10\hat{i} - 4\hat{j} - 11\hat{k},$
 $\overline{PM} \perp^r \bar{b}$
 $\therefore \overline{PM}. \bar{b} = 0$
 $\therefore \left[(9 + 10\lambda)\hat{i} + (-1 - 4\lambda) - 11(-13 - 11\lambda)\hat{k} \right]. \left(10\hat{i} - 4\hat{j} - 11\hat{k} \right) = 0$
 $\therefore 10(9 + 10\lambda) - 4(-1 - 4\lambda) - 11(13 - 11\lambda) = 0$
 $\therefore 237\lambda + 237 = 0$
 $\therefore \lambda = -1$

Putting this value of λ , we get the position vector of M as $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$. \therefore coordinates of the foot of perpendicular M are (1, 2, 3).

Now,
$$\overline{PM} = (\hat{i} + 2\hat{j} + 3\hat{k}) - (2\hat{i} - \hat{j} + 5\hat{k})$$

$$= -\hat{i} + 3\hat{j} - 2\hat{k}$$

$$\therefore |\overline{PM}| = \sqrt{(-1)^2 + (3)^2 + (-2)^2}$$

$$= \sqrt{1+9+4}$$

$$= \sqrt{14}$$

Hence, the coordinates of the foot of perpendicular are (1, 2, 3) and length of perpendicular = $\sqrt{14}$ units.

Exercise 6.2 | Q 3 | Page 207



Find the shortest distance between the lines

$$\overline{\mathbf{r}} = \left(4\hat{\mathrm{i}}-\hat{\mathrm{j}}
ight) + \lambda \left(\hat{\mathrm{i}}+2\hat{\mathrm{j}}-3\hat{\mathrm{k}}
ight) ext{ and } \overline{\mathbf{r}} = \left(\hat{\mathrm{i}}-\hat{\mathrm{j}}+2\hat{\mathrm{k}}
ight) + \mu \left(\hat{\mathrm{i}}+4\hat{\mathrm{j}}-5\hat{\mathrm{k}}
ight)$$

Solution:

We know that the shortest distance between the skew lines

 $\overline{\mathbf{r}} = \overline{\mathbf{a}}_1 + \lambda \overline{\mathbf{b}} \text{ and } \overline{\mathbf{r}} = \overline{\mathbf{a}}_2 + \mu \overline{\mathbf{b}}_2 \text{ is given by } \mathsf{d} = \frac{\left| (\overline{\mathbf{a}}_2 - \overline{\mathbf{a}}_1) . \left(\mathbf{b}_1 \times \mathbf{b}_2 \right) \right|}{\left| \overline{\mathbf{b}}_1 \times \overline{\mathbf{b}}_2 \right|}.$ Here, $\bar{a}_1 = 4\hat{i} - \hat{j}$, $\bar{a}_2 = \hat{i} - \hat{j} + 2\hat{k}$. $ar{
m b}_1 = \hat{
m i} + 2\hat{
m j} - 3\hat{
m k}, ar{
m b}_2 = \hat{
m i} + 4\hat{
m j} - 5\hat{
m k}$ $\dot{b}_1 imes ar{b}_2 = egin{bmatrix} \hat{i} & \hat{j} & \hat{k} \ 1 & 2 & -3 \ 1 & 4 & -5 \end{bmatrix}$ $= (-10+12)\hat{i} - (-5+3)\hat{j} + (4-2)\hat{k}$ $= 2\hat{i} + 2\hat{j} + 2\hat{k}$ and $ar{\mathrm{a}}_2-ar{\mathrm{a}}_1=\left(\hat{\mathrm{i}}-\hat{\mathrm{j}}+2\hat{\mathrm{k}}
ight)-\left(4\hat{\mathrm{i}}-\hat{\mathrm{j}}
ight)$ $= -3\hat{i} + 2\hat{k}$ $\dot{a}_2 = (\bar{\mathbf{a}}_2 - \bar{\mathbf{a}}_2). \left(ar{\mathbf{b}}_1 imes ar{\mathbf{b}}_2
ight) = \left(-3 \hat{\mathbf{i}} + 2 \hat{\mathbf{k}}
ight). \left(2 \hat{\mathbf{i}} + 2 \hat{\mathbf{j}} + 2 \hat{\mathbf{k}}
ight)$ = -3(2) + 0(2) + 2(2)= -6 + 0 + 4= -2 and $\left|\bar{\mathrm{b}}_1\times\bar{\mathrm{b}}_2\right|=\sqrt{2^2+2^2+2^2}$ $=\sqrt{4+4+4}$ $=2\sqrt{3}$



 \therefore required shortest distance between the given lines

$$= \left| \frac{-2}{2\sqrt{3}} \right|$$
$$= \frac{1}{\sqrt{3}}$$
 units.

Exercise 6.2 | Q 4 | Page 207

Find the shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} \text{ and } \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

Solution:

The shortest distance between the lines

$$\frac{x - x_1}{l_1} = \frac{y - y_1}{m_1} = \frac{z - z_1}{n_1} \text{ and } \frac{x - x_1}{l_2} = \frac{y - y_2}{m_{12}} = \frac{z - z_2}{n_2} \text{ is give n by}$$

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}$$

$$d = \frac{y - y_1 - y_2 - y_1 + (l_2 n_1 - 1_1 n_2)^2 + (l_1 m_2 - l_2 m_1)^2}{\sqrt{(m_1 n_2 - m_2 n_1)^2 + (l_2 n_1 - 1_1 n_2)^2 + (l_1 m_2 - l_2 m_1)^2}}$$
The equation of the given lines are
$$\frac{x + 1}{7} = \frac{y + 1}{-6} = \frac{z + 1}{1} \text{ and } \frac{x - 3}{1} = \frac{y - 5}{-2} = \frac{z - 7}{1}$$

$$\therefore x_1 = -1, y_1 = -1, z_1 = -1, x_2 = 3, y_2 = 5, z_2 = 7,$$

$$l_1 = 7, m_1 = -6, n_1 = 1, l_2 = 1, m_2 = -2, n_2 = 1$$

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = \begin{vmatrix} 4 & 6 & 8 \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}$$

$$= 4(-6 + 2) - 6(7 - 1) + 8(-14 + 6)$$

$$= -16 - 36 - 64$$

$$= -116$$



and

 $(m_1n_2 - m_2n_1)^2 + (l_2n_1 - l_1n_2)^2 + (l_1m_2 - l_2m_1)^2$ = (-6 + 2)2 + (1 - 7)2 + (1 - 7)2 + (-14 + 6) = 16 + 36 + 64 = 116

Hence, the required shortest distance between the given lines

$$= \left| \frac{-116}{\sqrt{116}} \right|$$
$$= \sqrt{116}$$
$$= 2\sqrt{29}$$
units.

Exercise 6.2 | Q 5 | Page 207

Find the perpendicular distance of the point (1, 0, 0) from the line $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ Also find the co-ordinates of the foot of the perpendicular.

Solution:

Let PM be the perpendicular drawn from the point (1, 0, 0) to the line $\frac{x+1}{2} = \frac{y-3}{-3} = \frac{z+10}{8} = \lambda$...(Say)

The coordinates of any point on the line are given by $x = -1 + 2\lambda$, $y = 3 + 3\lambda$, $z = 8 - \lambda$ Let the coordinates of M be

 $(-1 + 2\lambda, 3 + 3\lambda, -1 - \lambda) \qquad \dots(1)$ The direction ratios of PM are $-1 + 2\lambda - 2, 3 + 3\lambda + 3, -1 - \lambda - 1$ i.e. $2\lambda - 3, 3\lambda + 6, -\lambda - 2$ The direction ratios of the given line are 2, 3, 8. Since PM is perpendicular to the given line, we get $2(2\lambda - 3) + 3(3\lambda + 6) - 1(-\lambda - 2) = 0$ $\therefore 4\lambda - 6 + 9\lambda + 18 + \lambda + 2 = 0$ $\therefore 14\lambda + 14 = 0$



 $\therefore \lambda = -1.$ Put $\lambda = -1$ in (1), the coordinates of M are (-1 - 2, 3 - 3, -1 + 1) i.e. (-3, 0, 0). $\therefore \text{ length of perpendicular from P to the given line} = PM$

$$= \sqrt{(-3-2)^2 + (0+3)^2 + (0-1)^2}$$

= sqrt((25 + 9 + 1)
= $\sqrt{35}$ units.

Alternative Method :

We know that the perpendicular distance from the point

 $P|\overline{\infty}|$ to the lin $ar{\mathbf{r}}=ar{\mathbf{a}}+\lambdaar{\mathbf{b}}$ is given by

$$\begin{split} &\sqrt{\left|\overline{\infty}-\bar{a}\right|^{2}-\left[\frac{\left(\overline{00}-\bar{a}\right).\bar{b}}{\left|\bar{b}\right|}\right]^{2}} \qquad ...(1) \\ \text{Here, } \overline{\infty}=2\hat{i}-3\hat{j}+\hat{k}, \bar{a}=-\hat{i}+3\hat{j}-\hat{k}, \bar{b}=2\hat{i}+3\hat{j}-\hat{k} \\ \therefore \ \overline{\infty}-\bar{a}=\left(2\hat{i}-3\hat{j}+\hat{k}\right)-\left(-\hat{i}+3\hat{j}-\hat{k}\right) \\ =3\hat{i}-6\hat{j}+2\hat{k} \\ \therefore \ \left|\overline{\infty}-\bar{a}\right|^{2}=3^{2}+(-6)2+2^{2}=9+36+4=49 \\ \text{Also, } (\overline{\infty}-\bar{a}).\ \bar{b}=\left(3\hat{i}-6\hat{j}+2\hat{k}\right).\ \left(2\hat{i}+3\hat{j}-\hat{k}\right) \\ =(3)(2)+(-6)(3)+(2)(-1) \\ =6-18-2 \\ =-14 \\ \left|\bar{b}\right|=\sqrt{2^{2}+3^{2}+(-1)^{2}}=\sqrt{14} \end{split}$$

Substitutng tese values in (1), w get length of perpendicular from P to given line = PM



$$= \sqrt{49 - \left(-\frac{14}{\sqrt{14}}\right)^2}$$
$$= \sqrt{49 - 14}$$
$$= \sqrt{35}$$
units
or
 $2\sqrt{6}$ units, $(3, -4, 2)$.

Exercise 6.2 | Q 6 | Page 207

A(1, 0, 4), B(0, -11, 13), C(2, -3, 1) are three points and D is the foot of the perpendicular from A to BC. Find the co-ordinates of D.

Solution: Equation of the line passing through the points (x_{1}, y_{1}, z_{1}) and (x_{2}, y_{2}, z_{2}) is

$$rac{x-x_1}{x_2-x_1} = rac{y-y_1}{y_2-y_1} = rac{z-z_1}{z_2-z_1}$$

: the equation of the line BC passing through the points B (0, -11, 13) and C)2, -3,1) is

$$\frac{x-0}{2-0} = \frac{y+11}{-3+11} = \frac{z-13}{1-13}$$

i.e. $x(2) = \frac{y+11}{8} = \frac{z-13}{-12} = \lambda$...(Say)

AD is the perpendicular from the point A(1, 0, 4) to the line BC.

The coordinates of any point on the line BC are given by

x = 2 λ , y = -11 + 8 λ , z = 13 - 12 λ Let the coordinates of D be (2 λ , - 11 + 8 λ , 13 - 12 λ) ...(1) ∴ the direction ratio of AD are 2 λ - 1, λ 11 + 8 λ - 0, 13 - 12 λ - 4 i.e. 2 λ - 1, - 11 + 8 λ , 9 - 12 λ The direction ratios of the line BC are 2, 8, - 12. Since AD is perpendicular to BC, we get 2(2 λ - 1) + 8(- 11 + 8 λ) - 12(9 - 12 λ) = 0 ∴ 4 λ - 2 - 88 + 64 λ - 108 + 144 λ = 0 ∴ 212 λ - 198 = 0



$$\begin{split} &\therefore \lambda = \frac{198}{212} = \frac{99}{106} \\ &\text{Putting } \lambda = \frac{99}{106} \text{ in (1), the coordinates of D are} \\ &\left(\frac{198}{106}, -11 + \frac{792}{106}, 13 - \frac{1188}{106}\right) \\ &\text{i.e. } \left(\frac{198}{106}, \frac{-374}{106}, \frac{190}{106}\right), \\ &\text{i.e. } \left(\frac{99}{53}, \frac{-187}{53}, \frac{95}{53}\right). \end{split}$$

Exercise 6.2 | Q 7.1 | Page 207

By computing the shortest distance, determine whether following lines intersect each other.

$$\overline{\mathbf{r}} = \left(\hat{\mathrm{i}} - \hat{\mathrm{j}}
ight) + \lambda \left(2\hat{\mathrm{i}} + \hat{\mathrm{k}}
ight) ext{ and } \overline{\mathbf{r}} = \left(2\hat{\mathrm{i}} - \hat{\mathrm{j}}
ight) + \mu \left(\hat{\mathrm{i}} + \hat{\mathrm{j}} - \hat{\mathrm{k}}
ight)$$

Solution: The shortest distance between the lines

$$\begin{split} \bar{\mathbf{r}} &= \bar{\mathbf{a}}_1 + \lambda \bar{\mathbf{b}}_1 \text{ and } \bar{\mathbf{r}} = \bar{\mathbf{a}}_2 + \mu \bar{\mathbf{b}}_2 \text{ is given by} \\ d &= \left| \frac{(\bar{\mathbf{a}}_2 - \bar{\mathbf{a}}_1).(\bar{\mathbf{b}}_1 \times \bar{\mathbf{b}}_2)}{|\bar{\mathbf{b}}_1 \times \bar{\mathbf{b}}_2|} \right|. \\ \text{Here, } \bar{\mathbf{a}}_1 &= \hat{\mathbf{i}} - \hat{\mathbf{j}}, \bar{\mathbf{a}}_2 = 2\hat{\mathbf{i}} - \hat{\mathbf{j}}, \bar{\mathbf{b}}_1 = 2\hat{\mathbf{i}} + \hat{\mathbf{k}}, \bar{\mathbf{b}}_2 = \hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}. \\ \therefore \bar{\mathbf{b}}_1 \times \bar{\mathbf{b}}_2 &= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{k}} \\ 2 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix} \\ &= (0 - 1)\hat{\mathbf{i}} - (-2 - 1)\hat{\mathbf{j}} + (2 - 0)\hat{\mathbf{k}} \\ &= -\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}} \end{split}$$

And



$$\begin{split} \bar{a}_2 - \bar{a}_1 &= \left(2\hat{i} - \hat{j}\right) - \left(\hat{i} - \hat{j} - \hat{i}\right) \\ \therefore (\bar{a}_2 - \bar{a}_1). (\bar{b}_1 \times \bar{b}_2) &= \hat{i}. \left(-\hat{i} + 3\hat{j} + 2\hat{k}\right) \\ &= 1(-1) + 0(3) + 0(2) \\ &= -1 \\ \text{and} \end{split}$$

$$ig|ar{\mathbf{b}}_1 imes ar{\mathbf{b}}_2ig| = \sqrt{(-1)^2 + 3^2 + 2^2} = \sqrt{1 + 9 + 4} = \sqrt{14}$$

 \therefore the shortest distance between the given lines

$$= \left| \frac{-1}{\sqrt{14}} \right|$$
$$= \frac{1}{\sqrt{14}} \text{unit}$$

Hence, the given line do not intersect.

Exercise 6.2 | Q 7.2 | Page 207

By computing the shortest distance, determine whether following lines intersect each other.

$$\frac{x-5}{4} = \frac{y-7}{-5} = \frac{z+3}{-5}$$
 and $\frac{x-8}{7} = \frac{y-7}{1} = \frac{z-5}{3}$

Solution:

The shortest distance between the lines

$$\frac{x-5}{4} = \frac{y-7}{-5} = \frac{z+3}{-5} \text{ and } \frac{x-8}{7} = \frac{y-7}{1} = \frac{z-5}{3} \text{ is give n by}$$
$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}$$
$$d = \frac{\sqrt{(m_1n_2 - m_2n_1)^2 + (l_2n_1 - 1_1n_2)^2 + (l_1m_2 - l_2m_1)^2}}{\sqrt{(m_1n_2 - m_2n_1)^2 + (l_2n_1 - 1_1n_2)^2 + (l_1m_2 - l_2m_1)^2}}$$



The equation of the given lines are $\frac{x-5}{4} = \frac{y-7}{-5} = \frac{z+3}{-5} \text{ and } \frac{x-8}{7} = \frac{y-7}{1} = \frac{z-5}{3}$ $\therefore x_1 = -1, y_1 = -1, z_1 = -1, x_2 = 3, y_2 = 5, z_2 = 7,$ $l_1 = 7, m_1 = -6, n_1 = 1, l_2 = 1, m_2 = -2, n_2 = 1$ $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = \begin{vmatrix} 4 & 6 & 8 \\ 4 & -5 & -5 \\ 7 & 1 & 3 \end{vmatrix}$ = 4(-6+2) - 6(7-1) + 8(-14+6) = -16 - 36 - 64 = -116and $(m_{1}n_2 - m_{2}n_1)^2 + (l_{2}n_1 - l_{1}n_2)^2 + (l_{1}m_2 - l_{2}m_1)^2$ = (-6+2)2 + (1-7)2 + (1-7)2 + (-14+6) = 16 + 36 + 64 = 116

Hence, the required shortest distance between the given lines



or

The shortest distance between the lines = $\frac{282}{\sqrt{2820}}$ units

Hence, the given lines do not intersect.

Exercise 6.2 | Q 8 | Page 207



If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect each other, then find k. Solution: The lines $\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$ and $\frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$ intersect, if $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$ The equations of the given lines are $\frac{x-1}{2} = \frac{y+1}{2} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ \therefore x₁ = 1, y₁ = -1, z₁ = 1, x₂ = 3, y₂ = k, z₂ = 0, $l_1 = 2, m_1 = 3, n_1 = 4, l_2 = 1, m_2 = 2, n_2 = 1.$ Since these lines intersect, we get $\begin{vmatrix} 2 & k+1 & -1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$ $\therefore 2(3-8) - (k+1)(2-4) - 1(4-3) = 0$ $\therefore -10 + 2(k + 1) - 1 = 0$ $\therefore 2(k + 1) = 11$ ∴ k + 1 1/12 \therefore k = 9/2. EXERCISE 6.3 [PAGE 216]

Exercise 6.3 | Q 1 | Page 216

Find the vector equation of a plane which is at 42 unit distance from the origin and which is normal to the vector $2\hat{i}+\hat{j}-2\hat{k}$



Solution:

If $\hat{\mathbf{n}}$ is a unit vector along the normal and p i the length of the perpendicular from origin to the plane, then the vector equation of the plane $\bar{\mathbf{r}} \cdot \hat{\mathbf{n}} = p$ Here, $\bar{\mathbf{n}} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ and p = 42 $\therefore |\bar{\mathbf{n}}| = \sqrt{2^2 + 1^2 + (-2)^2}$ $= \sqrt{9}$ = 3 $\hat{\mathbf{n}} = \frac{\bar{\mathbf{n}}}{|\bar{\mathbf{n}}|}$ $= \frac{1}{3} \left(2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}} \right)$ \therefore th vector equation of the required plane is

$$\overline{\mathbf{r}} \cdot \left[\frac{1}{3} \left(2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}} \right) \right] = 42$$

i.e. $\overline{\mathbf{r}} \cdot \left(2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}} \right) = 126$.

Exercise 6.3 | Q 2 | Page 216

Find the perpendicular distance of the origin from the plane 6x - 2y + 3z - 7 = 0. **Solution:** The equation of the plane is

$$6x - 2y + 3z - 7 = 0$$

 \therefore its vector equation is

$$\overline{\mathbf{r}} \cdot \left(6\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}} \right) = 7$$
 ...(1)

where $ar{\mathbf{r}} = x \hat{\mathbf{i}} + y \hat{\mathbf{j}} + z \hat{\mathbf{k}}$

 $\therefore \bar{n} = 6 \hat{i} - 2 \hat{j} + 3 \hat{k}$ is normal to the plane.

$$|\bar{\mathbf{n}}| = \sqrt{6^2 + (-2)^2 + 3^2}$$

= $\sqrt{49}$
= 7



Unit vector along $\mathbf{\bar{n}}$ is

$$\hat{\mathrm{n}} = rac{ar{\mathrm{n}}}{|ar{\mathrm{n}}|} = rac{6\,\hat{\mathrm{i}}-2\,\hat{\mathrm{j}}+3\hat{\mathrm{k}}}{7}$$

Dividing bothsides of (1) by 7, we get

$$\overline{r}.\left(rac{6\widehat{i}-2\widehat{j}+3\widehat{k}}{7}
ight)=rac{7}{7}$$

Comparing with normal form of equation of the plane $\hat{\mathbf{r}} \cdot \hat{\mathbf{n}} = p$ it follows that length of perpendicular from origin is 1 unit.

Alternative Method :

The equation of the plane is 6x - 2y + 3z - 7 = 0

i.e.
$$\left(\frac{6}{6^2 + (-2)^2 + 3}\right)x - \left(\frac{2}{\sqrt{6^2} + (-2)^2 + 3^2}\right)y + \left(\frac{3}{\sqrt{6^2 + (-2)^2 + 3^2}}\right)z = \frac{7}{\sqrt{6^2 + (-2)^2 + 3^2}}$$

i.e. $\frac{6}{7}x - \frac{2}{7}y + \frac{3}{7}z = \frac{7}{7} = 1$

This is the normal form of the equation of plane.

 \therefore perpendicular distance of the origin frm the plane is p = 1 unit.

Exercise 6.3 | Q 3 | Page 216

Find the coordinates of the foot of the perpendicular drawn from the origin to the plane

2x + 6y - 3z = 63.

Solution:

The equation of the plane is 2x + 6y - 3z = 63.

Dividing each term by

$$\sqrt{2^{2} + 6^{2} + (-3)^{2}}$$

= $\sqrt{49}$
= 7,
we get
 $\frac{2}{7}x + \frac{6}{7}y - \frac{3}{7}z = \frac{63}{7} = 9$

This is the normal form of the equation of plane.



∴ the direction cosines of the perpendicular drawn from the origin to the plane are $I = \frac{2}{7}, m = \frac{6}{7}, n = -\frac{3}{7}$ and length of perpendicular from origin to the plane is p = 9. ∴ the coordinates of the foot of the perpendicular from the origin to the plane are (lp, mp, np)i.e. $\left(\frac{18}{7}, \frac{54}{7}, -\frac{27}{7}\right)$

Exercise 6.3 | Q 4 | Page 216

Reduce the equation $\overline{r}.\left(3\hat{i}+4\hat{j}+12\hat{k}
ight)$ to normal form and hence find

(i) the length of the perpendicular from the origin to the plane

(ii) direction cosines of the normal.

Solution:

The normal form of equation of a plane is $\mathbf{\bar{r}} \cdot \mathbf{\hat{n}} = p$ where $\mathbf{\hat{n}}$ is unit vector along the normal and p is the length of perpendicular drawn from origin to the plane.

Given pane is
$$\overline{\mathbf{r}}$$
. $\left(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 12\hat{\mathbf{k}}\right) = 78$...(1)

 $ar{\mathbf{n}}=3\hat{\mathbf{i}}+4\hat{\mathbf{j}}+12\hat{k}$ is normal to the plane

$$\therefore |\bar{\mathbf{n}}| = \sqrt{3^2 + 4^2 + 12^2} = \sqrt{169} = 13$$

Dividing both sides of (1) by 13, get

$$\bar{\mathbf{r}} \cdot \left(\frac{3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 12\hat{\mathbf{k}}}{13}\right) = \frac{78}{13}$$

i.e. $\bar{\mathbf{r}} \cdot \left(\frac{3}{13}\hat{\mathbf{i}} + \frac{4}{13}\hat{\mathbf{j}} + \frac{12}{13}\hat{\mathbf{k}}\right) = 6$

This is the normal form of the equation of plane.

Comparing with $\mathbf{\bar{r}} \cdot \hat{\mathbf{n}} = p$,

- (i) the length of the perpendicular from the origin to plane is 6.
- (ii) direction cosines of the normal are $\frac{3}{13}, \frac{4}{13}, \frac{12}{13}$.

Exercise 6.3 | Q 5 | Page 216



Find the vector equation of the plane passing through the point having position vector $\hat{i} + \hat{j} + \hat{k}$ and perpendicular to the vector $4\hat{i} + 5\hat{j} + 6\hat{k}$.

Solution:

The vector equation of the plane passing through the point $A(\bar{a})$ and perpendicular to the vector \bar{n} is $\bar{r} \cdot \bar{n} = \bar{a} \cdot \bar{n}$ Here.

$$\bar{\mathbf{a}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}},$$

$$\bar{\mathbf{n}} = 4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$$

$$\therefore \bar{\mathbf{a}} \cdot \bar{\mathbf{n}} = (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) \cdot (4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 6\hat{\mathbf{k}})$$

$$= (1)(4) + (1)(5) + (1)(6)$$

$$= 4 + 5 + 6$$

$$= 15$$

 \therefore the vector equation of the required plane is

$$\mathbf{\overline{r}}$$
. $\left(4\hat{\mathbf{i}}+5\hat{\mathbf{j}}+6\hat{\mathbf{k}}\right)$ = 15.

Exercise 6.3 | Q 6 | Page 216

Find the Cartesian equation of the plane passing through A(-1, 2, 3), the direction ratios of whose normal are 0, 2, 5.

Solution: The Cartesian equation of the plane passing through (x_1, y_1, z_1) , the direction

ratios of whose normal are a, b, c, is

 $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

 \div the cartesian equation of the required plane is

$$0(x + 1) + 2(y - 2) + 5(z - 3) = 0$$

i.e. 0 + 2y - 4 + 5z – 15 = 0

i.e. 2y + 5z = 19.

Exercise 6.3 | Q 7 | Page 216

Find the Cartesian equation of the plane passing through A(7, 8, 6) and parallel to the XY plane.



Solution: The Cartesian equation of the plane passing through (x_1, y_1, z_1) , the direction

ratios of whose normal are a, b, c, is

 $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

The required plane is parallel to XY-plane.

∴ it is perpendicular to Z-axis i.e. Z-axis is normal to the plane. Z-axis has direction ratios 0, 0, 1.

The plane passes through (7, 8, 6).

 \div the cartesian equation of the required plane is

0(x-7) + 0(y-8) + 1(z-6) = 0

i.e. z = 6.

Exercise 6.3 | Q 8 | Page 216

The foot of the perpendicular drawn from the origin to a plane is M(1,0,0). Find the vector equation of the plane.

Solution:

The vector equation of the plane passing through $A(\bar{a})$ and perpendicular to

 $\bar{\mathbf{n}} \operatorname{is} \bar{\mathbf{r}} \cdot \bar{\mathbf{n}} = \bar{\mathbf{a}} \cdot \bar{\mathbf{n}}$.

M(1,0,0) is the foot of the perpendicular drawn from origin to the plane. Then the plane is passing through M and is perpendicular to OM.

If $\overline{\mathbf{m}}$ is the position vector of M, then $\overline{\mathbf{m}} = \hat{\mathbf{i}}$.

Normal to the plane is

 $\mathbf{\bar{n}}=\overline{\mathbf{OM}}=\hat{\mathbf{i}}$

```
\overline{\mathbf{m}}.\,\overline{\mathbf{n}}=\hat{\mathbf{i}}.\,\hat{\mathbf{i}}=\mathbf{1}
```

.: the vector equation of the required plane is

r. î = 1.

Exercise 6.3 | Q 9 | Page 216

Find the vector equation of the plane passing through the point A(- 2, 7, 5) and parallel to vector $4\hat{i} - \hat{j} + 3\hat{k}$ and $\hat{i} - \hat{j} + \hat{k}$.



Solution:

The vector equation of the plane passing through the point $A(\bar{a})$ and parallel to the vectors \bar{b} and \bar{c} is

$$\bar{\mathbf{r}} \cdot (\bar{\mathbf{b}} \times \bar{\mathbf{c}}) = \bar{\mathbf{a}} \cdot (\bar{\mathbf{b}} \times \bar{\mathbf{c}}) \qquad \dots (1)$$
Here, $\bar{\mathbf{a}} = -2\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$
 $\bar{\mathbf{b}} = 4\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}},$
 $\bar{\mathbf{c}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$
 $\therefore \bar{\mathbf{b}} \times \bar{\mathbf{c}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 4 & -1 & 3 \\ 1 & 1 & 1 \end{vmatrix}$
 $= (-1 - 3)\hat{\mathbf{i}} - (4 - 3)\hat{\mathbf{j}} + (4 + 1)\hat{\mathbf{k}}$
 $= -4\hat{\mathbf{i}} - \hat{\mathbf{j}} + 5\hat{\mathbf{k}}$
 $\therefore \bar{\mathbf{a}} \cdot (\bar{\mathbf{b}} \times \bar{\mathbf{c}}) = (-2\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 5at\hat{\mathbf{k}}) \cdot (-4\hat{\mathbf{i}} - \hat{\mathbf{j}} + 5\hat{\mathbf{k}})$
 $= (-2)(-4) + (7)(-1) + (5)(5)$
 $= 8 - 7 + 2$
 $= 26$

 \therefore From (1), the vector equation of the required plane is $\mathbf{\bar{r}} \cdot \left(-4\hat{\mathbf{i}} - \hat{\mathbf{j}} + 5\hat{\mathbf{k}}\right) = 26.$

Exercise 6.3 | Q 10 | Page 216

Find the cartesian equation of the plane $\bar{\mathbf{r}} = \left(5\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}\right) + \lambda\left(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}\right) + \mu\left(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}\right).$

Solution:

The equation $\mathbf{\bar{r}} = \mathbf{\bar{a}} + \lambda \mathbf{\bar{b}} + \mu \mathbf{\bar{c}}$ represents a plane passing through a point having position vector $\mathbf{\bar{a}}$ and parallel to vectors $\mathbf{\bar{b}}$ and $\mathbf{\bar{c}}$.



```
Here,
```

$$\bar{\mathbf{a}} = 5\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}},$$

$$\bar{\mathbf{b}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}},$$

$$\bar{\mathbf{c}} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$$

$$\therefore \bar{\mathbf{b}} \times \bar{\mathbf{c}} = \begin{vmatrix} \hat{\mathbf{i}} & 1 & 1 \\ 1 & 1 & 2 & 3 \end{vmatrix}$$

$$= (3+2)\hat{\mathbf{i}} - (3-1)\hat{\mathbf{j}} + (-2-1)\hat{\mathbf{k}}$$

$$= 5\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$$

$$= \bar{\mathbf{a}}$$

Also,

$$\bar{\mathbf{a}}. (\bar{\mathbf{b}} \times \bar{\mathbf{c}})$$

$$= \bar{\mathbf{a}}. \bar{\mathbf{a}} = |\bar{\mathbf{a}}|^2$$

$$= (5)^2 + (-2)^2 + (3)^2$$

$$= 38$$

The vector equation of the plane passing through $A(\bar{a})$ and parallel to \bar{b} and \bar{c} is

$$ar{\mathbf{r}}.\left(ar{\mathbf{b}} imesar{\mathbf{c}}
ight)=ar{\mathbf{a}}.\left(ar{\mathbf{b}} imesar{\mathbf{c}}
ight)$$

 \therefore the vector equation of the given plane is

$$\overline{\mathbf{r}} \cdot \left(5\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}\right) = 38$$

If $\overline{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$, then this equation becomes
 $\left(x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}\right) \cdot \left(5\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}\right) = 38$
 $\therefore 5x - 2y - 3z = 38.$

This is the cartesian equation of the required plane.



Exercise 6.3 | Q 11 | Page 216

Find the vector equation of the plane which makes intercepts 1, 1, 1 on the co-ordinates axes.

Solution:

The vector equation of the plane passing through $\mathbf{A}(\bar{a}), \mathbf{B}(\bar{b})..\mathbf{C}(\bar{c})$, where A, B, C are non-collinear is $\mathbf{\bar{r}}.(\overline{\mathbf{AB}} \times \overline{\mathbf{AC}}) = \mathbf{\bar{a}}.(\overline{\mathbf{AB}} \times \overline{\mathbf{AC}})$...(1)

The required plane makes intercepts 1, 1, 1 on the coordinate axes.

: it passes through the three non-collinear points A = (1, 0, 0, B = (0, 1, 0), C = (0, 1, 1)

$$\therefore \bar{\mathbf{a}} = \hat{\mathbf{i}}, \bar{\mathbf{b}} = \hat{\mathbf{j}}, \bar{\mathbf{c}} = \hat{\mathbf{k}}$$

$$\overline{\mathbf{AB}} = \bar{\mathbf{b}} - \bar{\mathbf{a}} = \hat{\mathbf{j}} - \hat{\mathbf{i}} = -\hat{\mathbf{i}} + \hat{\mathbf{j}}$$

$$\therefore \overline{\mathbf{AC}} = \bar{\mathbf{c}} - \bar{\mathbf{a}} = \hat{\mathbf{k}} - \hat{\mathbf{i}} = -\hat{\mathbf{i}} + \hat{\mathbf{k}}$$

$$\therefore \overline{\mathbf{AB}} \times \overline{\mathbf{AC}} = \begin{vmatrix} \hat{\mathbf{i}} \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

$$= (1 - 0)\hat{\mathbf{i}} - (-1 - 0)\hat{\mathbf{j}} + (0 + 1)\hat{\mathbf{k}}$$

$$= \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$$
Also,
$$\bar{\mathbf{a}}. \left(\overline{\mathbf{AB}} \times \overline{\mathbf{AC}}\right)$$

$$= \hat{\mathbf{i}}. \left(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}\right)$$

$$= 1 \times 1 + 0 \times 1 + 0 \times 1$$

$$= 1$$

 \therefore from(1)the vector equation of the required plane is $\mathbf{\overline{r}} \cdot (\mathbf{\hat{i}} + \mathbf{\hat{j}} + hak) = 1$.

EXERCISE 6.4 [PAGE 220]

Exercise 6.4 | Q 1 | Page 220

Find the angle between planes r^{-} . $(i^+j^+2k^)=13$ and $r^{-}(2i^+j^+k^)=31$.



Solution:

Find the angle between planes $\mathbf{\bar{r}}$. $(\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = 13$ and $\mathbf{\bar{r}}(2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) = 31$.

The acute angle θ between the planes

 $\mathbf{\bar{r}}$. $\mathbf{\bar{n}}_1 = \mathbf{d}_1 \ \text{and} \ \mathbf{\bar{r}}$. $\mathbf{\bar{n}}_2 \mathbf{d}_2$ is given by $\cos \theta = \left| \frac{\bar{\mathbf{n}}_1 \cdot \bar{\mathbf{n}}_2}{|\bar{\mathbf{n}}_1| |\bar{\mathbf{n}}_2|} \right| \quad \dots(1)$ Here, $\bar{\mathbf{n}}_1 = \hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}},$ $ar{\mathbf{n}}_2 = 2 \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$ $\therefore \bar{n}_1 \cdot \bar{n}_2$ $= (\hat{i} + \hat{j} + 2\hat{k}).(2\hat{i} + \hat{j} + \hat{k})$ = (1)(2) + (1)(-1) + (2)(1)=2-1+2 = 3 Also, $|ar{\mathrm{n}}_1| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}$ $|ar{\mathrm{n}}_2| = \sqrt{2^2 + {(-1)}^2 + 1^2} = \sqrt{6}$: from (1), we have $\cos \theta = \left| \frac{3}{\sqrt{6}\sqrt{6}} \right|$ $=\frac{3}{6}$ $=\frac{1}{2}\cos 60^{\circ}$ $\therefore \theta = 60^{\circ}.$



Exercise 6.4 | Q 2 | Page 220

Find the acute angle between the line

$$\overline{r}.\left(\hat{i}+2\hat{j}+2\hat{k}\right)+\lambda\left(2\hat{i}+3\hat{j}-6\hat{k}\right) \text{and the plane } \overline{r}.\left(2\hat{i}-\hat{j}+\hat{k}\right)=0.$$

Solution:

The acute angle θ between the line $\mathbf{\bar{r}} = \mathbf{\bar{a}} + \lambda \mathbf{\bar{b}}$ and and the plane $\mathbf{\bar{r}} \cdot \mathbf{\bar{n}} = d$ is given by

$$\begin{aligned} \sin \theta &= \left| \frac{\bar{\mathbf{b}} \cdot \bar{\mathbf{n}}}{|\bar{\mathbf{b}}||\bar{\mathbf{n}}|} \right| \quad \dots(1) \\ \text{Here, } \bar{\mathbf{b}} &= 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 6\hat{\mathbf{k}}, \bar{\mathbf{n}} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}} \\ \therefore \bar{\mathbf{b}} \cdot \bar{\mathbf{n}} &= \left(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 6\hat{\mathbf{k}} \right) \cdot \left(2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}} \right) \\ &= (2)(2) + (3)(-1) + (-6)(1) \\ &= 4 - 3 - 6 \\ &= -5 \\ \text{Also, } \left| \bar{\mathbf{b}} \right| = \sqrt{2^2 + 3^2 + (-6)^2} = \sqrt{49} = 7 \\ &\left| \bar{\mathbf{n}} \right| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{6} \end{aligned}$$

: from (1), we have

$$\sin \theta = \left| \frac{-5}{7\sqrt{6}} \right| = \frac{5}{7\sqrt{6}}$$
$$\therefore \theta = \sin^{-1} \left(\frac{5}{7\sqrt{6}} \right).$$

Exercise 6.4 | Q 3 | Page 220

Show that the line $\bar{\mathbf{r}} = (2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) + \lambda(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$ and $\bar{\mathbf{r}} = (2\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) + \mu(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$ are coplanar. Find the equation of the plane determined by them.



Solution:

The lines
$$\bar{\mathbf{r}} = \bar{\mathbf{a}}_1 + \lambda_1 \bar{\mathbf{b}}_1$$
 and $\bar{\mathbf{r}} = \bar{\mathbf{a}}_2 + \lambda_2 \bar{\mathbf{b}}_2$ are coplanar if $\bar{\mathbf{a}}_1$. $(\bar{\mathbf{b}}_1 \times \bar{\mathbf{b}}_2) = \bar{\mathbf{a}}_2$. $(\bar{\mathbf{b}}_1 \times \bar{\mathbf{b}}_2)$
Here $\bar{\mathbf{a}}_1 = 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}, \bar{\mathbf{a}}_2 = 2\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$.
 $\bar{\mathbf{b}}_1 = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}, \bar{\mathbf{b}}_2 = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$
 $\therefore \bar{\mathbf{a}}_2 - \bar{\mathbf{a}}_1 = (2\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) - (2\hat{\mathbf{j}} - 3\hat{\mathbf{k}})$
 $= 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$
 $\bar{\mathbf{b}}_1 \times \bar{\mathbf{b}}_2 = \begin{vmatrix} \hat{\mathbf{i}} & \\ 1 & 2 & 3 \\ 2 & 3 \end{vmatrix}$
 $= (8 - 9)\hat{\mathbf{i}} - (4 - 6)\hat{\mathbf{j}} + (3 - 4)\hat{\mathbf{k}}$
 $= -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$
 $\therefore \bar{\mathbf{a}}_1$. $(\bar{\mathbf{b}}_1 \times \bar{\mathbf{b}}_2) = (2\hat{\mathbf{j}} - 3\hat{\mathbf{k}})$. $(-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}})$
 $= 0(-1) + 2(2) + (-3)(-1)$
 $= 0 + 4 + 3$
 $= 7$

$$\ddot{\mathbf{a}}_1.\left(ar{\mathrm{b}}_1 imesar{\mathrm{b}}_2
ight)=ar{\mathrm{a}}_2.\left(ar{\mathrm{b}}_1 imesar{\mathrm{b}}_2
ight)$$

Hence, the given lines are coplanar.

The plane determined by these lines is given by

Hence, the given lines are coplnar and the equation of the plane determined bt these lines is

$$\overline{\mathbf{r}} \cdot \left(-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}} \right) = 7.$$

Exercise 6.4 | Q 4 | Page 220

Find the distance of the point $4\hat{i} - 3\hat{j} + \hat{k}$ from the plane $\bar{r} \cdot \left(2\hat{i} + 3\hat{j} - 6\hat{k}\right)$ = 21.



Solution:

The distance of the point $A(\bar{a})$ from the plane $\bar{r} \cdot \bar{n} = p$ is given by $d = \frac{|\bar{a} \cdot \bar{n} - p|}{|\bar{n}|}$...(1)

Here,
$$\bar{\mathbf{a}} = 4\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}}, \bar{\mathbf{n}} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 6\hat{\mathbf{k}}, p = 21$$

 $\therefore \bar{\mathbf{a}}, \bar{\mathbf{n}} = (4\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}).(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 6\hat{\mathbf{k}})$
 $= (4)(2) + (-3)(3) + (1)(-6)$
 $= 8 - 9 - 6$
 $= -7$
Also, $|\bar{\mathbf{n}}| = \sqrt{2^2 + 3^2 + (-6)^2} = \sqrt{49} = 7$
 \therefore from (1), the required distance
 $= \frac{|-7 - 21|}{7}$

= 4 units.

Exercise 6.4 | Q 5 | Page 220

Find the distance of the point (1, 1 - 1) from the plane 3x + 4y - 12z + 20 = 0. Solution:

The distance of the point (x_1, y_1, z_1) from the plane ax + by + cz + d = 0 is

$$\left|\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}\right|$$

: the distance of the point (1, 1, -1) from the plane 3x + 4y - 12z + 20 = 0 is

$$\left| \frac{3(1) + 4(1 - 12(-1) + 20)}{\sqrt{3^2 + 4^2 + (-12)^2}} \right|$$

= $\left| \frac{3 + 4 + 12 + 20}{\sqrt{9 + 16 + 144}} \right|$
= $\frac{39}{\sqrt{169}}$
= $\frac{39}{13}$
= 3units.



MISCELLANEOUS EXERCISE 6 A [PAGES 207 - 209]

Miscellaneous Exercise 6 A | Q 1 | Page 207

Find the vector equation of the line passing through the point having position vector $3\hat{i} + 4\hat{j} - 7\hat{k}$ and parallel to $6\hat{i} - \hat{j} + \hat{k}$. Solution:

The vector equation of the line passing through A(\bar{a}) and parallel to the vector $\bar{\mathbf{b}}$ is $\bar{\mathbf{r}} = \bar{\mathbf{a}} + \lambda \bar{\mathbf{b}}$, where λ is a scalar.

 \therefore the vector equation of the line passing through the point having position vector

$$3\hat{i} + 4\hat{j} - 7\hat{k}$$
 and parallel to the vector $6\hat{i} - \hat{j} + \hat{k}$ is
 $\bar{r} = (3\hat{i} + 4\hat{j} - 7\hat{k}) + \lambda(6\hat{i} - \hat{j} + \hat{k}).$

Miscellaneous Exercise 6 A | Q 2 | Page 207

Find the vector equation of the line which passes through the point (3, 2, 1) and is parallel to the vector $2\hat{i} + 2\hat{j} - 3\hat{k}$.

Solution:

The vector equation of the line passing through A(\bar{a}) and parallel to the vector \bar{b} is $\bar{r} = \bar{a} + \lambda \bar{b}$, where λ is a scalar.

:. the vector equation of the line passing through the point having position vector $3\hat{i} + 2\hat{j} + \hat{k}$ and parallel to the vector $2\hat{i} + 2\hat{j} - 3\hat{k}$ is $\bar{r} = (3\hat{i} + 2\hat{j} + \hat{k}) + \lambda(2\hat{i} + 2\hat{j} - 3\hat{k})$.

Miscellaneous Exercise 6 A | Q 3 | Page 208

Find the Cartesian equations of the line which passes through the point (-2, 4, -5) and parallel to the line $\frac{x+2}{3} = \frac{y-3}{5} = \frac{z+5}{6}$.



Solution:

The line $\frac{x+2}{3} = \frac{y-3}{5} = \frac{z+5}{6}$ has direction ratios 3, 5, 6. The required line has direction ratios 3, 5, 6 as it is parallel to the given line.

It passes through the point (-2, 4, -5).

The cartesian equation of the line passing through (x_1, y_1, z_1) and

having direction ratios a, b, c are

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

 \therefore the required cartesian equation of the line are

$$\frac{x - (-2)}{3} = \frac{y - 4}{5} = \frac{z - (-5)}{6}$$

i.e.
$$\frac{x + 2}{3} = \frac{y - 4}{5} = \frac{z + 5}{6}.$$

Miscellaneous Exercise 6 A | Q 4 | Page 208

Obtain the vector equation of the line $\frac{x+5}{3} = \frac{y+4}{5} = \frac{z+5}{6}$.

Solution:

The cartesian equations of the line are $\frac{x+5}{3} = \frac{y+4}{5} = \frac{z+5}{6}$.

This line is passing through the point A(-5, -4, -5) and having direction ratios 3, 5, 6.

Let \bar{a} be the position vector of the point A w.r.t. the origin and \bar{b} be the vector parallel to the line.



Then
$$\bar{\mathbf{a}} = -5\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$$
 and $\bar{\mathbf{b}} = 3\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$.

The vector equation of the line passing through $A(\bar{a})$ and parallel to $\bar{b} is\bar{r} = \bar{a} + \lambda \bar{b}$ where λ is a scalar.

 \therefore the vector equation of the required line is

$$\overline{\mathbf{r}} = \left(-5\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 6\hat{\mathbf{k}}\right) + \lambda\left(3\hat{\mathbf{i}} + 5\hat{\mathbf{j}} + 6\hat{\mathbf{k}}\right).$$

Miscellaneous Exercise 6 A | Q 5 | Page 208

Find the vector equation of the line which passes through the origin and the point (5, -2, 3).

Solution:

Let $\overline{\mathbf{b}}$ be the position vector of the point B(5, -2, 3).

Then
$$ar{\mathrm{b}}=5\hat{\mathrm{i}}-2\hat{\mathrm{j}}+3\hat{\mathrm{k}}$$

Origin has position vector $\overline{0} = 0\hat{i} + 0\hat{j} + 0\hat{k}$.

The vector equation the line passing through

 $A(\bar{a}) \text{ and } b(\bar{b})is\bar{r} = \bar{a} + \lambda(\bar{b} - \bar{a})$ where λ is a scalar.

... the vector equation of the required line is

$$ar{\mathbf{r}} = ar{\mathbf{0}} + \lambda ig(ar{\mathbf{b}} - ar{\mathbf{0}} ig) = \lambda ig(5 \, \hat{\mathbf{i}} - 2 \, \hat{\mathbf{j}} + 3 \, \hat{\mathbf{k}} ig).$$

Miscellaneous Exercise 6 A | Q 6 | Page 208

Find the Cartesian equations of the line which passes through points (3, -2, -5) and (3, -2, 6).



Solution:

Let A = (3, -2, -5) and (3, -2, 6)

The direction ratios of the line AB are

3 - 3, - 2 - (- 2), 6 - (- 5) i.e. 0, 0, 11.

The parametric equations of the line passing through (x1, y1, z1)

and having direction ratios a, b, c are

x =
$$x_1 + a\lambda, y = y_1 \ b\lambda, z = z_1 + c\lambda$$

 \therefore The parametric equations of the line passing through (3, -2, -5)

and having direction ratios are 0, 0, 11 are

x = $3+(0)\lambda, y=-2+0(\lambda), z=-5+11\lambda$

i.e. x = 3, y = -2, $z = 11\lambda - 5$

 \therefore the cartesian equations of the line are

x = 3, y = -2, $z = 11\lambda - 5$, λ is a scalar.

Miscellaneous Exercise 6 A | Q 7 | Page 208

Find the Cartesian equations of the line passing through A(3, 2, 1) and B(1, 3, 1).

Solution: The direction ratios of the line AB are 3 - 1, 2 - 3, 1 - 1 i.e. 2, -1, 0. The parametric equations of the line passing through (x_1, y_1, z_1) and having direction ratios a, b, c are $x = x_1 + a\lambda$, $y = y_1 + b\lambda$, $z = z_1 + c\lambda$ \therefore the parametric equations of the line passing through (3, 2, 1) and having direction ratios 2, -1, 0 are $x = 3 + 2\lambda$, $y = 2 - \lambda$, $z = 1 + 0(\lambda)$

$$\therefore x - 3 = 2\lambda, y - 2 = -\lambda, z = 1$$

$$\therefore \frac{x-3}{2} = \frac{y-2}{-1} = \lambda, z = 1$$

 \therefore the cartesian equations of the required line are

$$\frac{x-3}{2} = \frac{y-2}{-1}, z = 1.$$



Miscellaneous Exercise 6 A | Q 8 | Page 208

Find the Cartesian equations of the line passing through the point

A(1, 1, 2) and perpendicular to the vectors

$$\bar{b}=\hat{i}+2\hat{j}+\hat{k} \ \text{and} \ \bar{c}=3\hat{i}+2\hat{j}-\hat{k}$$

Solution:

Let the required line have direction ratios p, q, r.

It is perpendicular to the vector

 $ar{\mathrm{b}}=\hat{\mathrm{i}}+2\hat{\mathrm{j}}+\hat{\mathrm{k}} \ \mathrm{and} \ ar{\mathrm{c}}=3\hat{\mathrm{i}}+2\hat{\mathrm{j}}-\hat{\mathrm{k}}.$

: it is perpendicular to lines whose direction ratios are 1, 2, 1 and 3, 2, -1.

$$\therefore p + 2q + r = 0, 3 + 2q - r = 0$$

$$\begin{array}{c} \therefore \ \frac{p}{\left|2 \quad 1\right|} = \frac{q}{\left|1 \quad 1\right|} = \frac{r}{\left|1 \quad 2\right|} \\ \frac{p}{\left|2 \quad -1\right|} = \frac{q}{\left|-1 \quad 3\right|} = \frac{r}{\left|3 \quad 2\right|} \\ \therefore \ \frac{p}{-4} = \frac{q}{4} = \frac{r}{-1} \\ \therefore \ \frac{p}{-1} = \frac{q}{1} = \frac{r}{-1} \end{array}$$

 \therefore the required line has direction ratios –1, 1, –1.

The cartesian equations of the line passing through (x₁, y₁, z₁) and having direction ratios a, b, c are $\frac{x = x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$

∴ the cartesian equation of the line passing through the point (1, 1, 2) and having directions ratios -1, 1, -1 are $\frac{x-1}{-1} = \frac{y-1}{1} = \frac{z-2}{-2}.$


Miscellaneous Exercise 6 A | Q 9 | Page 208

Find the Cartesian equations of the line which passes through the

point (2, 1, 3) and perpendicular to the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$.

Solution:

Let the required line have direction ratios p, q, r.

It is perpendicular to the vector

 $ar{\mathrm{b}} = \hat{\mathrm{i}} + 2\hat{\mathrm{j}} + \hat{\mathrm{k}} \ \mathrm{and} \ ar{\mathrm{c}} = 3\hat{\mathrm{i}} + 2\hat{\mathrm{j}} - \hat{\mathrm{k}}.$

 \therefore it is perpendicular to lines whose direction ratios are 1, 2, 1 and

3, 2, – 1.

$$\therefore \frac{p}{\begin{vmatrix} 2 & 1 \\ 2 & -1 \end{vmatrix}} = \frac{q}{\begin{vmatrix} 1 & 1 \\ -1 & 3 \end{vmatrix}} = \frac{r}{\begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix}$$
$$\therefore \frac{p}{-4} = \frac{q}{4} = \frac{r}{-1}$$
$$\therefore \frac{p}{2} = \frac{q}{-7} = \frac{r}{4}$$

 \therefore the required line has direction ratios 2, -7, 4.

The cartesian equations of the line passing through (x_1, y_1, z_1) and having direction ratios a, b, c are $\frac{x = x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$ \therefore the cartesian equation of the line passing through the point (2, -7, 4) and having directions ratios 2, -7, 4 are $\frac{x - 2}{2} = \frac{y - 1}{-7} = \frac{z - 2}{4}$.



+

Miscellaneous Exercise 6 A | Q 10 | Page 208

Find the vector equation of the line which passes through the origin and intersect the line x - 1 = y - 2 = z - 3 at right angle.

The given line is
$$rac{x-1}{1}=rac{y-2}{1}=rac{z-3}{1}=\lambda$$
 ...(Say)

 \therefore coordinates of any point on the line are x =

$$\lambda+1, y=\lambda+2, z=\lambda+3$$

∴ position vector of any point on the line is $(\lambda + 1)\hat{i} + (\lambda + 2)\hat{j} + (\lambda + 3)\hat{k}$...(1)

If $\mathbf{\bar{b}}$ is parallel to the given line whose direction ratios are 1, 1, 1 then $\mathbf{\bar{b}} = \mathbf{\hat{i}} + \mathbf{\hat{j}} + \mathbf{\hat{k}}$.

Let the required line passing through O meet the given line at M.

∴ position vector of M

=
$$\overline{\mathbf{m}} = (\lambda + 1)\hat{\mathbf{i}} + (\lambda + 2)\hat{\mathbf{j}} + (\lambda + 3)\hat{\mathbf{k}}$$
 ...[By (1)]

The required line is perpendicular to given line

$$\therefore \overline{OM}. \overline{b} = 0$$

$$\therefore \left[(\lambda + 1)\hat{i} + (\lambda + 2)\hat{j} + (\lambda + 3)\hat{k} \right]. (\hat{i} + \hat{j} + \hat{k}) = 0$$

$$\therefore (\lambda + 1) \times 1 + (\lambda + 2) \times 1 + (\lambda + 3) \times 1 = 0$$

$$\therefore 3\lambda + 6 = 0$$

$$\therefore \lambda = -2$$

$$\therefore \overline{m} = (-2 + 1)\hat{i} + (-2 + 2)\hat{j} + (-2 + 3)\hat{k} = -\hat{i} + \hat{k}$$



The vector equation of the line passing through

 $A(\bar{a}) \text{ and } B(\bar{b}) \text{ is } \bar{r} = \bar{a} + \lambda (\bar{b} - \bar{a}), \lambda \text{ is a scalar.}$

: the vector equation of the line passing through

 $O(\bar{0})$ and $M(\bar{m})is\bar{r} = \bar{0} + \lambda(\bar{m} - \bar{0}) = \lambda(-\hat{i} + \hat{k})$ where λ is a scalar.

Hence, vector equation of the required line is $ar{\mathbf{r}} = \lambda \Big(-\hat{\mathbf{i}} + \hat{\mathbf{k}} \Big)$.

Miscellaneous Exercise 6 A | Q 11 | Page 208

Find the value of λ so that the lines $\frac{1-x}{3} = \frac{7y-14}{\lambda} = \frac{z-3}{2} \text{ and } \frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$

are at right angles.

Solution:

The equations of the given lines are

$$\frac{1-x}{3} = \frac{7y-14}{2\lambda} = \frac{z-3}{2} \dots (1)$$

and
 $7-7x \qquad y-5 \qquad 6-z$

$$\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5} \quad ...(2)$$

Equation (1) can be written as:

$$\frac{-(x-1)}{3} = \frac{7(y-2)}{2\lambda} = \frac{z-3}{2}$$

i.e. $\frac{x-1}{-3} = \frac{y-2}{\frac{2\lambda}{7}} = \frac{z-3}{2}$

The direction ratios of this line are



$${
m a}_1=-3, b_1=rac{2\lambda}{7}, c_1=2$$

Equation (2) can be written as :

$$\frac{-7(x-1)}{3\lambda} = \frac{y-5}{1} = \frac{-(z-6)}{5}$$

i.e. $\frac{x-1}{-\frac{3\lambda}{7}} = \frac{y-5}{1} = \frac{z-6}{-5}$

The direction ratios of this line are

$$a_2=rac{-3\lambda}{7}, b_2=1, c_2=-5$$

Since the lines (1) and (2) are at right angles,

$$a_{1}a_{2} + b_{1}b_{2} + c_{1}c_{2} = 0$$

$$\therefore (-3)\left(\frac{-3\lambda}{7}\right) + \left(\frac{2\lambda}{7}\right)(1) + 2(-5) = 0$$

$$\therefore \left(\frac{9\lambda}{7}\right) + \left(\frac{2\lambda}{7}\right) - 10 = 0$$

$$\therefore \frac{11\lambda}{7} = 10$$

$$\therefore \lambda = \frac{70}{11}.$$

Miscellaneous Exercise 6 A | Q 12 | Page 208

Find the acute angle between the lines
$$\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-3}{2}$$
 and $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{1}$.



Solution:

Let
$$\bar{\mathbf{a}}$$
 and $\bar{\mathbf{b}}$ be the vectors in the direction of the lines

$$\frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-3}{2} \text{ and } \frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{1}$$
respectively.
Then $\bar{\mathbf{a}} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}, \bar{\mathbf{b}} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$
 $\therefore \bar{\mathbf{a}}, \bar{\mathbf{b}} = (\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}), (2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$
 $= (1)(2) + (-1)(1) + (2)(1)$
 $= 2 - 1 + 2$
 $= 3$
 $|\bar{\mathbf{a}}| = \sqrt{1^2 + (-1)^2 + 2^2} = \sqrt{6}$
 $|\bar{\mathbf{b}}| = \sqrt{2^2 + 1^2 + 1^2} = \sqrt{6}$
If θ is the angle between the lines, then
 $\cos \theta = \frac{\bar{\mathbf{a}}, \bar{\mathbf{b}}}{|\bar{\mathbf{a}}||\bar{\mathbf{b}}|} = \frac{3}{\sqrt{6}\sqrt{6}} = \frac{1}{2} = \cos 60^{\circ}$
 $\therefore \theta = 60^{\circ}.$

Miscellaneous Exercise 6 A | Q 13 | Page 208

Find the acute angle between the lines x = y, z = 0 and x = 0, z = 0.



Solution:

The equations x = y, z = 0 can be written as $\frac{x}{1} = \frac{y}{1}$, z = 0.

 \therefore the direction ratios of the line are 1, 1, 0.

The direction ratios of the line x = 0, z = 0, i.e., Y-axis are 0, 1, 0.

 \therefore its direction ratios are 0, 1, 0.

Let \bar{a} and \bar{b} be the vectors in the direction of the lines x = y, z = 0and x = 0, z = 0.

Then
$$\bar{\mathbf{a}} = \hat{\mathbf{i}} + \hat{\mathbf{j}}, \bar{\mathbf{b}} = \hat{\mathbf{j}}$$

 $\therefore \bar{\mathbf{a}}. \bar{\mathbf{b}} = (\hat{\mathbf{i}} + \hat{\mathbf{j}}). \hat{\mathbf{j}}$
 $= (1)(0) + (1)(1) + (0)(0)$
 $= 1$
 $|\bar{\mathbf{a}}| = \sqrt{1^2 + 1^2} = \sqrt{2}$
 $|\bar{\mathbf{b}}| = |\hat{\mathbf{j}}| = 1$

If θ is the acute angle between the lines, then

$$\cos \theta = \left| \frac{\bar{\mathbf{a}} \cdot \bar{\mathbf{b}}}{|\bar{\mathbf{a}}|\bar{\mathbf{b}}|} \right| = \left| \frac{1}{\sqrt{2} \times 1} \right| = \frac{1}{\sqrt{2}} = \cos 45^{\circ}.$$
$$\therefore \theta = 45^{\circ}.$$

Miscellaneous Exercise 6 A | Q 14 | Page 208

Find the acute angle between the lines x = -y, z = 0 and x = 0, z = 0.



Solution:

The equations x = – y, z = 0 can be written as $\frac{x}{1} = \frac{y}{1}$, z = 0.

 \therefore the direction ratios of the line are 1, 1, 0.

The direction ratios of the line x = 0, z = 0, i.e., Y-axis are 0, 1, 0.

 \therefore its direction ratios are 0, 1, 0.

Let $\bar{\mathbf{a}}$ and $\bar{\mathbf{b}}$ be the vectors in the direction of the lines x = y, z = 0 and x = 0, z = 0.

Then
$$\overline{\mathbf{a}} = \hat{\mathbf{i}} + \hat{\mathbf{j}}, \overline{\mathbf{b}} = \hat{\mathbf{j}}$$

 $\therefore \overline{\mathbf{a}}, \overline{\mathbf{b}} = (\hat{\mathbf{i}} + \hat{\mathbf{j}}), \hat{\mathbf{j}}$
 $= (1)(0) + (1)(1) + (0)(0)$
 $= 1$

$$\begin{aligned} |\bar{\mathbf{a}}| &= \sqrt{1^2 + 1^2} = \sqrt{2} \\ |\bar{\mathbf{b}}| &= \left| \hat{\mathbf{j}} \right| = 1 \end{aligned}$$

If $\boldsymbol{\theta}$ is the acute angle between the lines, then

$$\cos \theta = \left| \frac{\bar{\mathbf{a}} \cdot \bar{\mathbf{b}}}{|\bar{\mathbf{a}}|\bar{\mathbf{b}}|} \right| = \left| \frac{1}{\sqrt{2} \times 1} \right| = \frac{1}{\sqrt{2}} = \cos 45^{\circ}.$$
$$\therefore \theta = 45^{\circ}.$$

Miscellaneous Exercise 6 A | Q 15 | Page 208

Find the co-ordinates of the foot of the perpendicular drawn from the point (0, 2, 3) to the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$.

Solution: Let P = (0, 2, 3)

Let M be the foot of the perpendicular drawn from P to the line



$$rac{x+3}{5}=rac{y-1}{2}=rac{z+4}{3}=\lambda$$
 ...(Say)

The coordinates of any point on the line are given by $x = 5\lambda - 3$, $y = 2\lambda + 1$, $z = 3\lambda - 4$ Let $M = (5\lambda - 3, 2\lambda + 1, 3\lambda - 4)$...(1) The direction ratios of PM are $5\lambda - 3 - 0, 2\lambda + 1 - 2, 3\lambda - 4 - 3$ i.e. $5\lambda - 3\lambda, 2\lambda - 1, 3\lambda - 7$ Since, PM is perpendicular to the line whose direction ratios atr 5, 2, 3, $5(5\lambda - 3) + 2(2\lambda - 1) + 3(3\lambda - 7) = 0$ $\therefore 25\lambda - 15 + 4\lambda - 2 + 9\lambda - 21 = 0$ $\therefore 38\lambda - 38 = 0$ $\therefore \lambda = 1$ Substituting $\lambda = 1$ in (1), we get M = (5 - 3, 2 + 1, 3 - 4) = (2, 3, -1).Hence, the coordinates of the foot of perpendicular are (2, 3, -1).

Miscellaneous Exercise 6 A | Q 16.1 | Page 208

By computing the shortest distance determine whether following lines intersect each other :

$$\bar{\mathbf{r}} = \left(\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}\right) + \lambda \left(2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}\right) \text{ and } \bar{\mathbf{r}} \left(2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}\right) + \mu \left(\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}\right)$$

Solution: The shortest distance between the lines

$$\begin{split} \bar{\mathbf{r}} &= \bar{\mathbf{a}}_1 + \lambda \bar{\mathbf{b}}_1 \text{ and } \bar{\mathbf{r}} = \bar{\mathbf{a}}_2 + \mu \bar{\mathbf{b}}_2 \text{ is given by} \\ \mathsf{d} &= \left| \frac{(\bar{\mathbf{a}}_2 - \bar{\mathbf{a}}_1) \cdot (\bar{\mathbf{b}}_1 \times \bar{\mathbf{b}}_2)}{\left| \bar{\mathbf{b}}_1 \times \bar{\mathbf{b}}_2 \right|} \right|. \\ \mathsf{Here}, \, \bar{\mathbf{a}}_1 &= \hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}, \, \bar{\mathbf{a}}_2 = 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}, \\ \bar{\mathbf{b}}_1 &= 2\hat{\mathbf{i}} - \hat{\mathbf{i}} + \hat{\mathbf{k}}, \, \bar{\mathbf{b}}_2 = \hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}. \end{split}$$



$$\begin{split} \therefore \bar{b}_1 \times \bar{b}_2 &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 1 & -2 \end{vmatrix} \\ &= (2-1)\hat{i} - (-4-1)\hat{j} + (4+1)\hat{k} \\ &= \hat{i} - 5\hat{j} + 5\hat{k} \\ &\text{and} \\ \bar{a}_2 - \bar{a}_1 &= \left(2\hat{i} + 2\hat{j} - 3\hat{k}\right) - \left(\hat{i} + \hat{j} - \hat{k}\right) \\ &\therefore (\bar{a}_2 - \bar{a}_1). (\bar{b}_1 \times \bar{b}_2) = \hat{i}. \left(2\hat{i} + 2\hat{j} - 3\hat{k}\right) \\ &= 1(-1) + 0(3) + 0(2) \\ &= -1 \\ &\text{and} \\ &\left|\bar{b}_1 \times \bar{b}_2\right| = \sqrt{(-1)^2 + 3^2 + 2^2} \\ &= \sqrt{1+9+4} \\ &= \sqrt{14} \end{split}$$

Shortest distance between the lines is 0.

 \therefore the lines intersect each other.

Miscellaneous Exercise 6 A | Q 16.2 | Page 208

Bycomputing the shortest distance determine whether the fllowng line intersect ech other : $\frac{x-5}{4} = \frac{y-7}{5} = \frac{z+3}{5}$ and x - 6 = y - 8 = z + 2.



Solution: The shortest distance between the lines

$$\frac{x-5}{4} = \frac{y-7}{5} = \frac{z+3}{5} \text{ and } x-6 = y-8 = z+2 \text{ is given by}$$

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}$$

$$d = \frac{x-5}{\sqrt{(m_1n_2 - m_2n_1)^2 + (l_2n_1 - l_1n_2)^2 + (l_1m_2 - l_2m_1)^2}}{\sqrt{(m_1n_2 - m_2n_1)^2 + (l_2n_1 - l_1n_2)^2 + (l_1m_2 - l_2m_1)^2}}$$
The equation of the given lines are
$$\frac{x-5}{4} = \frac{y-7}{5} = \frac{z+3}{5} \text{ and } x-6 = y-8 = z+2$$

$$\therefore x_1 = 5, y_1 = 7, z_1 = 3, x_2 = 6, y_2 = 8, z_2 = 2,$$

$$l_1 = 4, m_1 = 5, n_1 = 1, l_2 = 1, m_2 = -2, n_2 = 1$$

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = \begin{vmatrix} 4 & 6 & 8 \\ 4 & 5 & 5 \\ -6 & -8 & 2 \end{vmatrix}$$

$$= 4(-6+2)-6(7-1)+8(-14+6)$$

$$= -16-36-64$$

$$= -116$$
and
$$(m_{1n_2} - m_{2n_1})^2 + (l_{2n_1} - l_{2n_2})^2 + (l_{1m_2} - l_{2m_1})^2$$

$$= (-6+2)^2 + (1-7)^2 + (1-7)^2 + (-14+6)$$

$$= 16+36+64$$

$$= 116$$

Hence, the required shortest distance between the given lines



$$= \left| \frac{-116}{\sqrt{116}} \right|$$

$$=\sqrt{116}$$

 $= 2\sqrt{29}$ units

or

Shortest distance between the lines is 0.

 \therefore the lines intersect each other.

Miscellaneous Exercise 6 A | Q 17 | Page 208

If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-2}{1} = \frac{y+m}{2} = \frac{z-2}{1}$

intersect each other, find m.

Solution:

The lines
$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$$
 and $\frac{x-2}{1} = \frac{y+m}{2} = \frac{z-2}{1}$ intersect, if

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_2 & z - z_2 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0 \qquad ...(1)$$
Here, $(x_1, y_1, z_1) \equiv (1, -1, 1)$,
 $(x_2, y_2, z_2) \equiv (2, -m, 2)$,

 $(x_2, y_2, z_2) = (z, -11, z),$ $a_1 = 2, b_1 = 3, c_1 = 4,$ $a_2 = 1, b_2 = 2, c_2 = 1$ Substituting these values in (1), we get



$$\begin{vmatrix} 2-1 & -m+1 & 2-1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & 1-m & 1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$$\therefore 1(3-8) - (1-m)(2-4) + 1(4-3) = 0$$

$$\therefore -5 + 2 - 2m + 1 = 0$$

$$\therefore -2m = 2$$

$$\therefore m = -1.$$

Miscellaneous Exercise 6 A | Q 18 | Page 208

Find the vector and Cartesian equations of the line passing through the point (-1, -1, 2) and parallel to the line 2x - 2 = 3y + 1 = 6z - 2. Solution:

Let \bar{a} be the person vector of the point A(-1, -1, 2) w.r.t. the origin.

Then $ar{\mathrm{a}} = - \hat{\mathrm{i}} - \hat{\mathrm{j}} + 2 \hat{\mathrm{k}}$

The equation of given line is

$$2x - 2 = 3y + 1 = 6z - 2$$

$$\therefore 2(x - 1) = 3\left(y + \frac{1}{3}\right) = 6\left(z - \frac{1}{3}\right)$$

$$\therefore \frac{x - 1}{\left(\frac{1}{2}\right)} = \frac{y + \frac{1}{3}}{\left(\frac{1}{3}\right)} = \frac{z - \frac{1}{3}}{\left(\frac{1}{6}\right)}$$

The direction ratios of this line are



$$rac{1}{2},rac{1}{3},rac{1}{6}$$
 i.e. 3, 2, 1

Let $\bar{\mathbf{b}}$ be the vector parallel to this line.

Then
$$ar{\mathrm{b}}=3 \, \hat{\mathrm{i}}+2 \, \hat{\mathrm{j}}+\hat{\mathrm{k}}$$

The vector equation of the line passing through $A(\bar{a})$ and parallel to \bar{b} is

 $ar{\mathbf{r}} = ar{\mathbf{a}} + \lambda ar{\mathbf{b}}$, where λ is a scalar

 \therefore the vector equation of the required line is

$$\overline{\mathbf{r}} = \left(-\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}\right) + \lambda\left(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}\right).$$

The line passes through (-1, -1, 2) and has direction ratios 3, 2, 1

 \therefore the cartesian equations of the line are

$$\frac{x - (-1)}{3} = \frac{y - (-1)}{2} = \frac{z - 2}{1}$$

i.e.
$$\frac{x + 1}{3} = \frac{y + 1}{2} = \frac{z - 2}{1}.$$

Miscellaneous Exercise 6 A | Q 19 | Page 208

Find the direction cosines of the lines

$$\overline{\mathbf{r}} = \left(-2\hat{\mathbf{i}} + \frac{5}{2}\hat{\mathbf{j}} - \hat{\mathbf{k}}\right) + \lambda\left(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}\right).$$

Solution:

The line
$$ar{\mathbf{r}} = \left(-2\hat{\mathbf{i}} + rac{5}{2}\hat{\mathbf{j}} - \hat{\mathbf{k}}
ight) + \lambda\left(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}
ight)$$
 is parallel to $ar{\mathbf{b}} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$.

 \therefore direction ratios of the line are 2, 3, 0



 \therefore direction cosines of the line are

$$\frac{2}{\sqrt{2^2+3^2+0}}, \frac{3}{\sqrt{2^2+3^2+0}}, 0$$

i.e. $\frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}}, 0.$

Miscellaneous Exercise 6 A | Q 20 | Page 208

Find the Cartesian equation of the line passing through the origin which is perpendicular to x - 1 = y - 2 = z - 1 and intersect the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$.

Solution: Let the required line have direction ratios a, b, c Since the line passes through the origin, its cartesian equation are

$$\frac{x}{a} = \frac{y}{b} = \frac{z}{c} \qquad \dots (1)$$

This line is perpendicular to the line

$$x - 1 = y - 2 = z - 1 \text{ whose direction ratios are 1, 1, 1}$$

∴ a + b + c = 0 ...(2)
The lines $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_2} = \frac{z - z_1}{c_1}$ intersect, if

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

Applying this condition for the lines

$$\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$$
 and $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ we get



$$\begin{vmatrix} 1-0 & -1-0 & 1-0 \\ a & b & c \\ 2 & 3 & 4 \end{vmatrix} = 0$$

$$\therefore 1(4b - 3c) + 1(4a - 2c) + 1(3a - 2b) = 0$$

$$\therefore 4b - 3c + 4a - 2c + 3a - 2b = 0$$

$$\therefore 7a + 2b - 5c = 0 \qquad ...(3)$$

From (2) and (3), we get

$$\frac{a}{\begin{vmatrix} 1 & 1 \\ 2 & -5 \end{vmatrix}} = \frac{b}{\begin{vmatrix} 1 & 1 \\ -5 & 7 \end{vmatrix}} = \frac{a}{\begin{vmatrix} 1 & 1 \\ 7 & 2 \end{vmatrix}$$
$$\therefore \frac{a}{-7} = \frac{b}{12} = \frac{c}{-5}$$

 \therefore the required line has direction ratios –7, 12, –5.

From (1), cartesian equation of required line are

$$\frac{x}{-7} = \frac{y}{12} = \frac{z}{-5}$$

i.e. $\frac{x}{7} = \frac{y}{-12} = \frac{z}{5}$

Miscellaneous Exercise 6 A | Q 21 | Page 208

Find the vector equation of the line whose Cartesian equations are y = 2 and 4x - 3z + 5 = 0.

Solution: 4x - 3z + 5 = 0 can be written as

$$4x = 3z - 5 = 3\left(z - \frac{5}{3}\right)$$
$$\therefore \frac{4x}{12} = \frac{3\left(z - \frac{5}{3}\right)}{12}$$



$$\therefore \frac{x}{3} = \frac{z - \frac{5}{3}}{4}$$

 \therefore the cartesian equation of the line are

$$\frac{x}{3} = \frac{z - \frac{5}{3}}{4}, y = 2.$$

This line passes through the point $A\left(0,2,\frac{5}{3}\right)$ whose position vector is $\bar{a} = 2\hat{j} + \frac{5}{3}\hat{k}$

Also the line has direction ratio 3, 0, 4.

If $ar{\mathrm{b}}$ is a vector parallel to the line, then $ar{\mathrm{b}}=3\hat{\mathrm{i}}+4\hat{\mathrm{k}}$

The vector equation of the line pasing through $A(\bar{a})$ and parallel to \bar{b} is $\bar{r} = \bar{a} + \lambda \bar{b}$ where λ is a scalar.

... the vector equation of the required line is

$$\bar{\mathbf{r}} = \left(2\hat{\mathbf{j}} + \frac{5}{3}\hat{\mathbf{k}}\right) + \lambda\left(3\hat{\mathbf{i}} + 4\hat{\mathbf{k}}\right).$$

Miscellaneous Exercise 6 A | Q 22 | Page 209

Find the coordinates of points on th line $\frac{x-1}{1} = \frac{y-2}{-2} = \frac{z-3}{2}$ which are at the distance 3 unit from the base point A(I, 2, 3).

Solution:

The cartesian equations of the line are $rac{x-1}{1}=rac{y-2}{-2}=rac{z-3}{2}=\lambda$...(Say)



The coordinates of any point on this line are given by

 $x = \lambda + 1y = -2\lambda + 2z = 2\lambda + 3$

Let M (λ + 1, -2 λ + 2, 2 λ + 3) ...(1)

be the point on the in whose dstance from A(1, 2, 3) is 3 units.

$$\therefore \sqrt{\lambda + 1 - 1^2 + (-2\lambda + 2 - 2)^2 + (2\lambda + 3 - 3)^2} \therefore \sqrt{\lambda^2 + 4\lambda^2 + 4\lambda^2} = 3 \therefore \sqrt{9\lambda^2} = 3 \therefore 9\lambda^2 = 9 \therefore \lambda = \pm 1 When \lambda = 1, M = (1 + 1, -2 + 2, 2 + 3) ...[By (1)] i.e. M = (2, 0, 5) When \lambda = -1, M = (1 - 1, 2 + 2, -2 + 3) ...[By (1)] i.e. M = (0, 4, 1)$$

Hence, the coordinates of the required points are (2, 0, 5) and (0, 4, 1).

MISCELLANEOUS EXERCISE 6 B [PAGES 223 - 225]

Miscellaneous Exercise 6 B | Q 1 | Page 223

Choose correct alternatives :

If the line
$$\frac{x}{3} = \frac{y}{4} = z$$
 is perpendicular to the line
 $\frac{x-1}{k} = \frac{y+2}{3} = \frac{z-3}{k-1}$, then the value of k is
1. 11/4
2. -11/4
3. 11/2
4. 4/11
Solution: -11/4

Miscellaneous Exercise 6 B | Q 2 | Page 223



Choose correct alternatives :

The vector equation of line 2x - 1 = 3y + 2 = z - 2 is Options

$$\begin{split} \bar{\mathbf{r}} &= \left(\frac{1}{2}\hat{\mathbf{i}} - \frac{2}{3}\hat{\mathbf{j}} + 2\hat{\mathbf{k}}\right) + \lambda\left(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 6\hat{\mathbf{k}}\right)\\ \bar{\mathbf{r}} &= \hat{\mathbf{i}} - \hat{\mathbf{j}} + \left(2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}\right)\\ \bar{\mathbf{r}} &= \left(\frac{1}{2}\hat{\mathbf{i}} - \hat{\mathbf{j}}\right) + \lambda\left(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 6\hat{\mathbf{k}}\right)\\ \bar{\mathbf{r}} &= \left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right) + \lambda\left(\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 6\hat{\mathbf{k}}\right) \end{split}$$

Solution:

$$\overline{\mathbf{r}} = \left(\frac{1}{2}\hat{\mathbf{i}} - \frac{2}{3}\hat{\mathbf{j}} + 2\hat{\mathbf{k}}\right) + \lambda\left(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 6\hat{\mathbf{k}}\right)$$

Miscellaneous Exercise 6 B | Q 3 | Page 223

Choose correct alternatives :

The direction ratios of the line which is perpendicular to the two lines $\frac{x-7}{2} = \frac{y+17}{-3} = \frac{z-6}{1} \text{ and } \frac{x+5}{1} = \frac{y+3}{2} = \frac{z-6}{-2} \text{ are}$ 1. 4, 5, 7 2. 4, -5, 7 3. 4, -5, -7 4. -4, 5, 8 Solution: 4, 5, 7

Miscellaneous Exercise 6 B | Q 4 | Page 223



Choose correct alternatives :

The length of the perpendicular from (1, 6,3) to the line

 $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ 1. 3
2. $\sqrt{11}$ 3. $\sqrt{13}$ 4. 5
Solution: $\sqrt{13}$

Miscellaneous Exercise 6 B | Q 5 | Page 224

Choose correct alternatives :

The shortest distance between the lines

$$\bar{\mathbf{r}} = \left(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}\right) + \lambda \left(\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}\right) \text{ and } \bar{\mathbf{r}} = \left(2\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}\right) + \mu \left(2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}\right) \text{ is }$$

- 1. 1/√3
- 2. 1/√2
- **3.** 3/√2
- 4. √3/2

Solution: $3/\sqrt{2}$

Miscellaneous Exercise 6 B | Q 6 | Page 224

Choose correct alternatives :

The lines

$$\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k} \text{ and } \frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1} \text{ are coplnar if}$$
1. k = 1 or -1
2. k = 0 or -3
3. k = ± 3
4. k = 0 or -1



Solution: k = 0 or – 3

Miscellaneous Exercise 6 B | Q 7 | Page 224

Choose correct alternatives :

The lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x-1}{-2} = \frac{y-2}{-4} = \frac{z-3}{6}$ are

- 1. perpendicular
- 2. intersecting
- 3. skew
- 4. coincident

Solution: intesecting

Miscellaneous Exercise 6 B | Q 8 | Page 224

Choose correct alternatives :

Equation of X-axis is

- 1. x = y = z
- 2. y = z
- 3. y = 0, z = 0
- 4. x = 0, y = 0

Solution: y = 0, z = 0

Miscellaneous Exercise 6 B | Q 9 | Page 224

Choose correct alternatives :

The angle between the lines 2x = 3y = -z and 6x = -y = -4z is

- 1. 45°
- 2. 30°
- 3. 0°
- **4. 90°**

Solution: 90°

Miscellaneous Exercise 6 B | Q 10 | Page 224

Choose correct alternatives :



Te direction ratios of the line 3x + 1 = 6y - 2 = 1 - z are

- 1. 2, 1, 6
- 2. 2, 1, -6
- 3. 2, -1, 6
- 4. -2, 1, 6

Solution: 2, 1, – 6

Miscellaneous Exercise 6 B | Q 11 | Page 224

Choose correct alternatives :

The perpendicular distance of the plane 2x + 3y - z = k from the origin is $\sqrt{14}$ units, the value of k is

- 1. 14
- 2. 196
- 3. 2√14
- 4. √14/2

Solution: 14

Miscellaneous Exercise 6 B | Q 12 | Page 224

Choose correct alternatives :

The angle between the planes

$$\overline{\mathbf{r}}.\left(\hat{\mathbf{i}}-2\hat{\mathbf{j}}+3\hat{\mathbf{k}}\right)+4=0 \ \mathrm{and} \ \overline{\mathbf{r}}.\left(2\hat{\mathbf{i}}+\hat{\mathbf{j}}-3\hat{\mathbf{k}}\right)+7=0 \ \mathrm{is}$$

Options

 $\frac{\pi}{2}$ $\frac{\pi}{3}$ $\cos^{-1}\left(\frac{3}{4}\right)$ $\cos^{-1}\left(\frac{9}{14}\right)$



Solution:

$$\cos^{-1}\left(rac{9}{14}
ight)$$

Miscellaneous Exercise 6 B | Q 13 | Page 224

Choose correct alternatives :

If the planes
$$\bar{\mathbf{r}} \cdot \left(2\hat{\mathbf{i}} - \lambda\hat{\mathbf{j}} + \hat{\mathbf{k}}\right) = 3 \; \mathrm{and} \; \bar{\mathbf{r}} \cdot \left(4\hat{\mathbf{i}} - \hat{\mathbf{j}} + \mu\hat{\mathbf{k}}\right) = 5$$

are parallel, then the values of λ and μ are respectively

Options



Solution:

$$\frac{1}{2}, 2$$

Miscellaneous Exercise 6 B | Q 14 | Page 225

Choose correct alternatives :

The equation of the plane passing through (2, -1, 3) and making equal intercepts on the coordinate axes is

- 1. x + y + z = 1
- 2. x + y + z = 2
- 3. x + y + z = 3
- 4. x + y + z = 4



Solution: x + y + z = 4

Miscellaneous Exercise 6 B | Q 15 | Page 225

Choose correct alternatives :

Measure of angle between the plane 5x - 2y + 3z - 7 = 0 and 15x - 6y + 9z + 5 = 0 is

- **1. 0°**
- 2. 30°
- 3. 45°
- 4. 90°

Solution: 0°

Miscellaneous Exercise 6 B | Q 16 | Page 225

Choose correct alternatives :

The direction cosines of the normal to the plane 2x - y + 2z = 3 are

Options

$$\frac{\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}}{\frac{-2}{3}, \frac{1}{3}, \frac{-2}{3}}{\frac{2}{3}, \frac{1}{3}, \frac{2}{3}}{\frac{2}{3}, \frac{1}{3}, \frac{2}{3}}{\frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}}$$

Solution:

 $\frac{2}{3}, \frac{-1}{3}, \frac{2}{3}$

Miscellaneous Exercise 6 B | Q 17 | Page 225

Choose correct alternatives :



The equation of the plane passing through the points (1, -1, 1), (3, 2, 4) and parallel to Y-axis is :

- 1. 3x + 2z 1 = 0
- 2. 3x 2z = 1
- 3. 3x + 2z + 1 = 0
- 4. 3x + 2z = 2

Solution: 3x - 2z = 1

Miscellaneous Exercise 6 B | Q 18 | Page 225

Choose correct alternatives :

The equation of the plane in which the line $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5} \text{ and } \frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3} \text{ lie, is}$ 1. 17x - 47y - 24z + 172 = 02. 17x + 47y - 24z + 172 = 03. 17x + 47y + 24z + 172 = 04. 17x - 47y + 24z + 172 = 0Solution: 17x - 47y - 24z + 172 = 0

Miscellaneous Exercise 6 B | Q 19 | Page 225

If the line $\frac{x+1}{2} = \frac{y-m}{3} = \frac{z-4}{6}$ lies in the plane 3x - 14y + 6z + 49 = 0, then the value of m is 1. 5 2. 3 3. 2

4. - 5

Solution: 5

Miscellaneous Exercise 6 B | Q 20 | Page 225

Choose correct alternatives :

The foot of perpendicular drawn from the point (0,0,0) to the plane is (4, -2, -5) then the equation of the plane is



1. 4x + y + 5z = 14

2.
$$4x - 2y - 5z = 45$$

- 3. x 2y 5z = 10
- 4. 4x + y + 6z = 11

Solution: 4x - 2y - 5z = 45.

MISCELLANEOUS EXERCISE 6 B [PAGES 225 - 226]

Miscellaneous Exercise 6 B | Q 1 | Page 225

Solve the following :

Find the vector equation of the plane which is at a distance of 5 units from the origin and which is normal to the vector

 $2\hat{i} + \hat{j} + 2\hat{k}$.

Solution:

If $\hat{\mathbf{n}}$ is a unit vector along the normal and p i the length of the perpendicular from origin to the plane, then the vector equation of the plane $\mathbf{\bar{r}} \cdot \hat{\mathbf{n}} = p$

Here,
$$\bar{\mathbf{n}} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$$
 and $\mathbf{p} = 5$

$$\therefore |\bar{\mathbf{n}}| = \sqrt{2^2 + 1^2 + (2)^2}$$

$$= \sqrt{9}$$

$$= 3$$

$$\hat{\mathbf{n}} = \frac{\bar{\mathbf{n}}}{|\bar{\mathbf{n}}|}$$

$$= \frac{1}{3} \left(2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}} \right)$$

 \therefore the vector equation of the required plane is

$$\overline{\mathbf{r}} \cdot \left[\frac{1}{3} \left(2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}} \right) \right] = 5$$

i.e.
$$\overline{\mathbf{r}} \cdot \left(2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}} \right) = 15.$$



Miscellaneous Exercise 6 B | Q 2 | Page 225

Solve the following :

Find the perpendicular distance of the origin from the plane 6x + 2y + 3z - 7 = 0**Solution:**

The distance of the point (x_1, y_1, z_1) from the plane ax + by + cz +

d = 0 is
$$\left| \frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}} \right|$$

: the distance of the point (1, 1, – 1) from the plane 6x + 2y + 3z - 7 = 0 is

$$\left| \frac{6(1) + 2(1 - 3(-1) + 7)}{\sqrt{3^2 + 4^2 + (-12)^2}} \right|$$
$$= \left| \frac{6 + 4 + 6 + 7}{\sqrt{9 + 16 + 144}} \right|$$
$$= \frac{23}{\sqrt{169}}$$
$$= \frac{23}{13}$$
$$= 1 \text{ units.}$$

Miscellaneous Exercise 6 B | Q 3 | Page 225

Solve the following :

Find the coordinates of the foot of the perpendicular drawn from the origin to the plane 2x + 3y + 6z = 49.

Solution: The equation of the plane is 2x + 3y + 6z = 49.

Dividing each term by



$$\sqrt{2^{2} + 3^{2} + (-6)^{2}}$$

= $\sqrt{49}$
= 7,
we get
 $\frac{2}{7}x + \frac{3}{7}y - \frac{6}{7}z = \frac{49}{7} = 7$

This is the normal form of the equation of plane.

 \therefore the direction cosines of the perpendicular drawn from the origin to the plane are

$$|=\frac{2}{7}, m=\frac{3}{7}, n=\frac{6}{7}$$

and length of perpendicular from origin to the plane is p = 7.

 \div the coordinates of the foot of the perpendicular from the origin to the plane are

$$(lp, mp, np)$$
i.e. $(2, 3, 6)$

Miscellaneous Exercise 6 B | Q 4 | Page 225

Solve the following :

Reduce the equation $ar{\mathbf{r}}.\left(6\hat{\mathbf{i}}+8\hat{\mathbf{j}}+24\hat{\mathbf{k}}
ight)$ = 13 normal form and

hence find

- (i) the length of the perpendicular from the origin to the plane.
- (ii) direction cosines of the normal.

Solution:

The normal form of equation of a plane is $\bar{\mathbf{r}} \cdot \hat{\mathbf{n}} = p$ where $\hat{\mathbf{n}}$ is unit vector along the normal and p is the length of perpendicular drawn from origin to the plane.



Given pane is $\bar{\mathbf{r}} \cdot \left(6\hat{\mathbf{i}} + 8\hat{\mathbf{j}} + 24\hat{\mathbf{k}}\right) = 13$...(1) $\bar{\mathbf{n}} = 6\hat{\mathbf{i}} + 8\hat{\mathbf{j}} + 24\hat{\mathbf{k}}$ is normal to the plane $\therefore |\bar{\mathbf{n}}| = \sqrt{6^2 + 8^2 + 24^2} = \sqrt{76} = 13$ Dividing both sides of (1) by 13, get $\bar{\mathbf{r}} \cdot \left(\frac{3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 12\hat{\mathbf{k}}}{13}\right) = \frac{76}{13}$ i.e. $\bar{\mathbf{r}} \cdot \left(\frac{3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 12\hat{\mathbf{k}}}{13}\hat{\mathbf{j}} + \frac{12}{13}\hat{\mathbf{k}}\right) = \frac{1}{2}$

This is the normal form of the equation of plane.

Comparing with $\mathbf{\bar{r}} \cdot \hat{\mathbf{n}} = p_{i}$

(i) the length of the perpendicular from the origin to plane is $\frac{1}{2}$.

(ii) direction cosines of the normal are $\frac{3}{13}, \frac{4}{13}, \frac{12}{13}$.

Miscellaneous Exercise 6 B | Q 5 | Page 226

Solve the following :

Find the vector equation of the plane passing through the points A(1, 92, 1), B(2, 91, 93) and C(0, 1, 5).

Solution: The vector equation of the plane passing through three non-collinear points

$$A(\bar{a}), B(\bar{b}) \text{ and } C(\bar{c}) \text{ is}\bar{r}. (\overline{AB} \times \overline{AC}) = \bar{a}. (\overline{AB} \times \overline{AC})$$
...(1)

Here, $ar{a}=\hat{i}-2\hat{j}+\hat{k}, ar{b}=2\hat{i}-\hat{j}-3\hat{k}, ar{c}=\hat{j}+5\hat{k}$



$$\therefore \overline{AB} = \overline{b} - \overline{a} = \left(2\hat{i} - \hat{j} - 3\hat{k}\right) - \left(\hat{i} - 2\hat{j} + \hat{k}\right)$$

$$= \hat{i} + \hat{j} - 4\hat{k}$$

$$\overline{AC} = \overline{c} - \overline{a} = \left(\hat{j} + 5\hat{k}\right) - \left(\hat{i} - 2\hat{j} + \hat{k}\right)$$

$$= \hat{i} + 3\hat{j} + 4\hat{k}$$

$$\therefore \overline{AB} \times \overline{AC} = \begin{vmatrix} \hat{i} & \hat{k} \\ 1 & 1 & -4 \\ -1 & 3 & 4 \end{vmatrix}$$

$$= (4 + 12)\hat{i} - (4 - 4)\hat{j} + (3 + 1)\hat{k}$$

$$= 16\hat{i} + 4\hat{k}$$

$$\text{Now, } \overline{a}. \left(\overline{AB} \times \overline{AC}\right) = \left(\hat{i} - 2\hat{j} + \hat{k}\right). \left(16\hat{i} + 4\hat{k}\right)$$

$$= (1)(16) + (-2)(0) + (1)(4) = 20$$

$$\therefore \text{ from(1), the vector equation of the required plane is }$$

$$\overline{r}. \left(16\hat{i} + 4\hat{k}\right) = 20.$$

Miscellaneous Exercise 6 B | Q 6 | Page 226

Solve the following :

Find the cartesian equation of the plane passing through A(1,-2, 3) and direction ratios of whose normal are 0, 2, 0.

Solution: The Cartesian equation of the plane passing through (x_1, y_1, z_1) , the direction ratios of whose normal are a, b, c, is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$ \therefore the cartesian equation of the required plane is 0(x + 1) + 2(y + 2) + 5(z - 3) = 0



i.e. 0 + 2y - 4 + 10z - 15 = 0 i.e. y + 2 = 0.

Miscellaneous Exercise 6 B | Q 7 | Page 226

Solve the following :

Find the cartesian equation of the plane passing through A(7, 8, 6) and parallel to the plane $\bar{\mathbf{r}} \cdot \left(6\hat{\mathbf{i}} + 8\hat{\mathbf{j}} + 7\hat{\mathbf{k}}\right) = 0$.

Solution: The cartesian equation of the plane

$$\bar{r}.\left(6\hat{i}+8\hat{j}+7\hat{k}\right) = 0$$
 is 6x + 8y + 7z = 0

The required plane is parallel to it \therefore its cartesian equation is 6x + 8y + 7z = p ...(1)

A(7, 8, 6) lies on it and hence satisfies its equation

 \therefore (6)(7) + (8)(8) + (7)(6) = p

i.e., p = 42 + 64 + 42 = 148.

: from (1), the cartesian equation of the required plane is 6x + 8y + 7z = 148.

Miscellaneous Exercise 6 B | Q 8 | Page 226

Solve the following :

The foot of the perpendicular drawn from the origin to a plane is M(1, 2, 0). Find the vector equation of the plane.

Solution:

The vector equation of the plane passing through $A(\bar{a})$ and

perpendicular to $\mathbf{\bar{n}}$ is $\mathbf{\bar{r}}$. $\mathbf{\bar{n}} = \mathbf{\bar{a}}$. $\mathbf{\bar{n}}$.

M(1, 2, 0) is the foot of the perpendicular drawn from origin to the

plane. Then the plane is passing through M and is perpendicular to OM.

If $\overline{\mathbf{m}}$ is the position vector of M, then $\overline{\mathbf{m}} = \hat{\mathbf{i}}$.



Normal to the plane is

$$\bar{\mathbf{n}} = \overline{\mathbf{OM}} = \hat{\mathbf{i}}$$

 $\overline{\mathbf{m}}.\,\overline{\mathbf{n}} = \hat{\mathbf{i}}.\,\hat{\mathbf{i}} = 5$

 \therefore the vector equation of the required plane is

$$\mathbf{\bar{r}}.\left(\hat{\mathbf{i}}+2\hat{\mathbf{j}}\right)$$
 = 5.

Miscellaneous Exercise 6 B | Q 9 | Page 226

Solve the following :

A plane makes non zero intercepts a, b, c on the coordinate axes.

Show that the vector equation of the plane is

$$\mathbf{\bar{r}} \cdot \left(bc\hat{\mathbf{i}} + ca\hat{\mathbf{j}} + ab\hat{\mathbf{k}} \right)$$
 = abc.

Solution:

The vector equation of the plane passing through $A(\bar{a}), B(\bar{b})..C(\bar{c})$, where A, B, C are non-collinear is $\bar{r}.(\overline{AB} \times \overline{AC}) = \bar{a}.(\overline{AB} \times \overline{AC})$...(1)

The required plane makes intercepts 1, 1, 1 on the coordinate axes.

: it passes through the three non-collinear points A =(1, 0, 0, B = (0, 1, 0), C = (0, , 1)



$$\begin{split} &\therefore \bar{\mathbf{a}} = \hat{\mathbf{i}}, \bar{\mathbf{b}} = \hat{\mathbf{j}}, \bar{\mathbf{c}} = \hat{\mathbf{k}} \\ \overline{\mathbf{AB}} = \bar{\mathbf{b}} - \bar{\mathbf{a}} = \hat{\mathbf{j}} - \hat{\mathbf{i}} = -\hat{\mathbf{i}} + \hat{\mathbf{j}} \\ &\therefore \overline{\mathbf{AC}} = \bar{\mathbf{c}} - \bar{\mathbf{a}} = \hat{\mathbf{k}} - \hat{\mathbf{i}} = -\hat{\mathbf{i}} + \hat{\mathbf{k}} \\ &\therefore \overline{\mathbf{AB}} \times \overline{\mathbf{AC}} = \begin{vmatrix} \hat{\mathbf{i}} \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} \\ &= (1 - 0)\hat{\mathbf{i}} - (-1 - 0)\hat{\mathbf{j}} + (0 + 1)\hat{\mathbf{k}} \\ &= \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}} \\ &\text{Also,} \\ &\bar{\mathbf{a}}. \left(\overline{\mathbf{AB}} \times \overline{\mathbf{AC}}\right) \\ &= \hat{\mathbf{i}}. \left(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}\right) \end{split}$$

$$= 1 \times 1 + 0 \times 1 + 0 \times 1$$

= 1

 \therefore from(1)the vector equation of the required plane is

$$\overline{\mathbf{r}}.\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}\right)=\mathbf{1}.$$

Miscellaneous Exercise 6 B | Q 10 | Page 226

Solve the following :

Find the vector equation of the plane passing through the point A(- 2, 3, 5) and parallel to the vectors $4\hat{i} + 3\hat{k}$ and $\hat{i} + \hat{j}$.

Solution:

The vector equation of the plane passing through the point $A(\bar{a})$ and parallel to the vectors \bar{b} and \bar{c} is

$$\mathbf{\bar{r}}.(\mathbf{\bar{b}}\times\mathbf{\bar{c}})=\mathbf{\bar{a}}.(\mathbf{\bar{b}}\times\mathbf{\bar{c}})$$
 ...(1)



Here,
$$\bar{a} = -2\hat{i} + 3\hat{j} + 5\hat{k}$$

 $\bar{b} = 4\hat{i} + 3\hat{k}$,
 $\bar{c} = \hat{i} + \hat{j}$
 $\therefore \bar{b} \times \bar{c} = \begin{vmatrix} \hat{i} & \hat{k} \\ 4 & 0 & 3 \\ 1 & 1 & 0 \end{vmatrix}$
 $= (-1 - 3)\hat{i} - (4 - 3)\hat{j} + (3 + 1)\hat{k}$
 $= -4\hat{i} - \hat{j} + 4\hat{k}$
 $\therefore \bar{a} \cdot (\bar{b} \times \bar{c}) = (-2\hat{i} + 7\hat{j} + 5\hat{k}) \cdot (-4\hat{i} - \hat{j} + 4\hat{k})$
 $= (-2)(-4) + (7)(-1) + (5)(4)$
 $= 8 - 7 + 8$
 $= 35$

 \therefore From (1), the vector equation of the required plane is

$$\mathbf{\bar{r}} \cdot \left(-3\hat{\mathbf{i}} - 3at\mathbf{j} + 4\hat{\mathbf{k}}\right) = 35.$$

Miscellaneous Exercise 6 B | Q 11 | Page 226

Solve the following :

Find the cartesian equation of the plane

$$\bar{\mathbf{r}} = \lambda \left(\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}\right) + \mu \left(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}\right).$$

Solution:

The equation $\mathbf{\bar{r}} = \mathbf{\bar{a}} + \lambda \mathbf{\bar{b}} + \mu \mathbf{\bar{c}}$ represents a plane passing through a point having position vector $\mathbf{\bar{a}}$ and parallel to vectors $\mathbf{\bar{b}}$ and $\mathbf{\bar{c}}$.



Here, $\bar{\mathbf{b}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}},$ $\bar{\mathbf{c}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ $\therefore \bar{\mathbf{b}} \times \bar{\mathbf{c}} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix}$ $= (3+2)\hat{\mathbf{i}} - (3-1)\hat{\mathbf{j}} + (-2-1)\hat{\mathbf{k}}$ $= 5\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - \hat{\mathbf{k}}$ $= \bar{\mathbf{a}}$ Also, $\bar{\mathbf{a}}. (\bar{\mathbf{b}} \times \bar{\mathbf{c}})$ $= \bar{\mathbf{a}}. \bar{\mathbf{a}} = |\bar{\mathbf{a}}|^2$ $= (5)^2 + (4)^2 + (0)^2$ = 0

The vector equation of the plane passing through A(\bar{a}) and parallel to \bar{b} and \bar{c} is

$$ar{\mathbf{r}}.\left(ar{\mathbf{b}} imesar{\mathbf{c}}
ight)=ar{\mathbf{a}}.\left(ar{\mathbf{b}} imesar{\mathbf{c}}
ight)$$

 \therefore the vector equation of the given plane is

$$\overline{\mathbf{r}} \cdot \left(5\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - \hat{\mathbf{k}}\right) = 0$$

If $ar{{f r}}=x \, {f {f i}}+y \, {f {f j}}+z {f {f k}}$, then this equation becomes

$$\left(x\hat{\mathbf{i}}+y\hat{\mathbf{j}}+z\hat{\mathbf{k}}\right).\left(5\hat{\mathbf{i}}-4\hat{\mathbf{j}}-\hat{\mathbf{k}}\right)=0$$

$$\therefore 5x - 4y + z = 0.$$

This is the cartesian equation of the required plane.



Miscellaneous Exercise 6 B | Q 12 | Page 226

Solve the following :

Find the cartesian equations of the planes which pass through A(1, 2, 3), B(3, 2, 1) and make equal intercepts on the coordinate axes.

Solution: Case 1 : Let all the intercepts be 0.

Then the plane passes through the origin.

Then the cartesian equation of the plane is ax + by + cz = 0. ...(1)

(1, 2, 3) d (3, 2, 1) lie on the plane.

a + 2b + 3c = 0 and 3a + 2b + c = 0



∴ a, b, c are proprtional to 1, – 2, 1 ∴ from (1), the required cartesian equation is x - 2y + z = 0

Case 2 : Let he plane make non zero intercept p on each axis.

then its equation is
$$\frac{x}{p} + \frac{y}{p} + \frac{z}{p}$$
 = 1

i.e. x + y + z = p ...(2) Since this plane pass through (1, 2, 3) and (3, 2, 1)

 \therefore 1 + 2 + 3 = p and 3 + 2 + 1 = p

∴ p = 6

:. from (2), the required cartesian equation is x + y + z = 6Hence, the cartesian equations of required planes are x + y + z = 6 and x - 2y + z = 0.

Miscellaneous Exercise 6 B | Q 13 | Page 226



Solve the following :

Find the vector equation of the plane which makes equal non zero intercepts on the coordinate axes and passes through (1, 1, 1).

Solution: Case 1 : Let all the intercepts be 0.

Then the plane passes through the origin.

Then the vector equation of the plane is ax + by + cz = 0. ...(1)

(1, 1, 1) lie on the plane.

$$\therefore \frac{\hat{i}}{\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}} = \frac{\hat{j}}{\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}} = \frac{\hat{k}}{\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}}$$
$$\therefore \frac{\hat{i}}{1} = \frac{\hat{j}}{1} = \frac{\hat{k}}{1}$$
i.e. $\frac{\hat{i}}{1} = \frac{\hat{j}}{1} = \frac{\hat{k}}{1}$

 $\therefore \, \hat{i}, \hat{j}, \hat{k}$ are proprtional to 1, 1, 1

 \therefore from (1), the required cartesian equation is x - y + z = 0

Case 2 : Let he plane make non zero intercept p on each axis.

then its equation is
$$\frac{\hat{i}}{p} + \frac{\hat{j}}{p} + \frac{\hat{k}}{p} = 1$$

i.e. $\hat{i} + \hat{j} + \hat{k} = p$...(2)
Since this plane pass through (1, 1, 1)
 $\therefore 1 + 1 + 1 = p$
 $\therefore p = 3$


:. from (2), the required cartesian equation is $\hat{i} + \hat{j} + \hat{k} = 3$ Hence, the cartesian equations of required planes are $\bar{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 3$

Miscellaneous Exercise 6 B | Q 14 | Page 226

Solve the following :

Find the angle between the planes $\mathbf{\bar{r}} \cdot \left(-2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}\right) = 17$ and $\mathbf{\bar{r}} \cdot \left(2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}\right) = 71$.

Solution:

The acute angle between the planes

$$\begin{split} \bar{\mathbf{r}} \cdot \bar{\mathbf{n}}_1 &= d_1 \text{ and } \bar{\mathbf{r}} \cdot \bar{\mathbf{n}}_2 = d_2 \text{ is given by} \\ \cos \theta &= \left| \frac{\bar{\mathbf{n}}_1 \cdot \bar{\mathbf{n}}_2}{|\bar{\mathbf{n}}_1||\bar{\mathbf{n}}_2|} \right| \quad \dots(1) \\ \text{Here,} \\ \bar{\mathbf{n}}_1 &= -2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}}, \\ \bar{\mathbf{n}}_2 &= 2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}} \\ \therefore \bar{\mathbf{n}}_1 \cdot \bar{\mathbf{n}}_2 \\ &= \left(2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}} \right) \cdot \left(2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}} \right) \\ &= (1)(2) + (1)(1) + (2)(1) \\ = 2 + 1 + 2 \\ = 5 \\ \text{Also,} \end{split}$$



 $\begin{aligned} |\bar{\mathbf{n}}_1| &= \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6} \\ |\bar{\mathbf{n}}_2| &= \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{6} \\ \therefore \text{ from (1), we have} \\ \cos \theta &= \left| \frac{3}{\sqrt{6}\sqrt{6}} \right| \\ &= \frac{3}{6} \\ &= \frac{1}{2} \cos 90^\circ \\ \therefore \theta = 90^\circ. \end{aligned}$

Miscellaneous Exercise 6 B | Q 15 | Page 226

Solve the following :

Find the acute angle between the line $\bar{\mathbf{r}} = \lambda \left(\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}} \right)$ and the plane $\bar{\mathbf{r}} \cdot \left(2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}} \right) = 23$.

Solution:

The acute angle θ between the line $\bar{\mathbf{r}} = \bar{\mathbf{a}} + \lambda \bar{\mathbf{b}}$ and and the plane $\bar{\mathbf{r}} \cdot \bar{\mathbf{n}} = d$ is given by

$$\sin \theta = \left| \frac{\mathbf{\bar{b}} \cdot \mathbf{\bar{n}}}{\left| \mathbf{\bar{b}} \right| \left| \mathbf{\bar{n}} \right|} \right| \qquad \dots (1)$$

Here, $\bar{\mathbf{b}}=\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{k}, \bar{\mathbf{n}}=2\hat{\mathbf{i}}-\hat{\mathbf{j}}+\hat{k}$



$$\therefore \bar{\mathbf{b}} \cdot \bar{\mathbf{n}} = \left(\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}\right) \cdot \left(2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}\right)$$

$$= (2)(2) + (3)(-1) + (-6)(1)$$

$$= 4 - 3 - 6$$

$$= -5$$
Also, $|\bar{\mathbf{b}}| = \sqrt{1^2 + 1^2 + (-1)^2} = \sqrt{2} = 1$
 $|\bar{\mathbf{n}}| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{4}$
 \therefore from (1), we have
 $\sin \theta = \left|\frac{2\sqrt{2}}{-3}\right| = \frac{2\sqrt{2}}{3}$
 $\therefore \theta = \sin^{-1}\left(\frac{2\sqrt{2}}{3}\right).$

Miscellaneous Exercise 6 B | Q 16 | Page 226

Show that the line $\mathbf{\bar{r}} = (2\hat{j} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$ and $\mathbf{\bar{r}} = (2\hat{i} + 6\hat{j} + 3\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 4\hat{k})$ are coplanar. Find the equation of the plane determined by them.

Solution:

The lines
$$\bar{\mathbf{r}} = \bar{\mathbf{a}}_1 + \lambda_1 \bar{\mathbf{b}}_1$$
 and $\bar{\mathbf{r}} = \bar{\mathbf{a}}_2 + \lambda_2 \bar{\mathbf{b}}_2$ are coplanar If
 $\bar{\mathbf{a}}_1 \cdot (\bar{\mathbf{b}}_1 \times \bar{\mathbf{b}}_2) = \bar{\mathbf{a}}_2 \cdot (\bar{\mathbf{b}}_1 \times \bar{\mathbf{b}}_2)$
Here $\bar{\mathbf{a}}_1 = 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}, \bar{\mathbf{a}}_2 = 2\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 3\hat{\mathbf{k}},$
 $\bar{\mathbf{b}}_1 = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}, \bar{\mathbf{b}}_2 = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$
 $\therefore \bar{\mathbf{a}}_2 - \bar{\mathbf{a}}_1 = (2\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) - (2\hat{\mathbf{j}} - 3\hat{\mathbf{k}})$
 $= 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$



$$\begin{split} \bar{\mathbf{b}}_{1} \times \bar{\mathbf{b}}_{2} &= \begin{vmatrix} \hat{\mathbf{i}} & & \\ 1 & 2 & 3 \\ 2 & 3 \end{vmatrix} \\ &= (8 - 9)\hat{\mathbf{i}} - (4 - 6)\hat{\mathbf{j}} + (3 - 4)\hat{\mathbf{k}} \\ &= -\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}} \\ &\therefore \bar{\mathbf{a}}_{1} \cdot (\bar{\mathbf{b}}_{1} \times \bar{\mathbf{b}}_{2}) = (2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}) \cdot (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}) \\ &= 0(-1) + 2(2) + (-3)(-1) \\ &= 0 + 4 + 3 \\ &= 7 \\ &\text{and } \bar{\mathbf{a}}_{2} \cdot (\bar{\mathbf{b}}_{1} \times \bar{\mathbf{b}}_{2}) = (2\hat{\mathbf{j}} + 6\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) \cdot (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}) \\ &= 2(-1) + 6(2) + 3(-1) \\ &= -2 + 12 - 3 \\ &= 7 \\ &\therefore \bar{\mathbf{a}}_{1} \cdot (\bar{\mathbf{b}}_{1} \times \bar{\mathbf{b}}_{2}) = \bar{\mathbf{a}}_{2} \cdot (\bar{\mathbf{b}}_{1} \times \bar{\mathbf{b}}_{2}) \end{split}$$

Hence, the given lines are coplanar.

The plane determined by these lines is given by

$$\therefore \mathbf{\bar{r}} \cdot \left(\mathbf{\bar{b}}_1 \times \mathbf{\bar{b}}_2 \right) = \mathbf{\bar{a}}_1 \cdot \left(\mathbf{\bar{b}}_1 \times \mathbf{\bar{b}}_2 \right)$$

i.e. $\mathbf{\bar{r}} \cdot \left(-\mathbf{\hat{i}} + 2\mathbf{\hat{j}} - \mathbf{\hat{k}} \right) = 7$

Hence, the given lines are coplnar and the equation of the plane determined bt these lines is

$$\overline{\mathbf{r}} \cdot \left(-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}\right) = 7.$$



Miscellaneous Exercise 6 B | Q 17 | Page 226

Solve the following :

Find the distance of the point $3\hat{i} + 3\hat{j} + \hat{k}$ from the plane $\bar{r} \cdot (2\hat{i} + 3\hat{j} + 6\hat{k}) = 21$.

Solution:

The distance of the point A(\bar{a}) from the plane $\bar{\mathbf{r}} \cdot \bar{\mathbf{n}} = p$ is given by $d = \frac{|\bar{\mathbf{a}} \cdot \bar{\mathbf{n}} - p|}{|\bar{\mathbf{n}}|}$...(1) Here, $\bar{\mathbf{a}} = 3\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}, \bar{\mathbf{n}} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 6\hat{\mathbf{k}}, p = 21$ $\therefore \bar{\mathbf{a}} \cdot \bar{\mathbf{n}} = (3\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}) \cdot (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 6\hat{\mathbf{k}})$ = (3)(2) + (3)(3) + (1)(-6) = 6 + 6 - 6 = 6Also, $|\bar{\mathbf{n}}| = \sqrt{3^2 + 3^2 + (-6)^2} = \sqrt{-12} = 0$ \therefore from (1), the required distance

$$=\frac{|-12-21|}{12}$$

= 0 units.

Miscellaneous Exercise 6 B | Q 18 | Page 226

Solve the following :

Find the distance of the point (13, 13, -13) from the plane 3x + 4y - 12z = 0.



Solution:

The distance of the point (x_1, y_1, z_1) from the plane ax + by + cz +

d = 0 is
$$\left| \frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}} \right|$$

: the distance of the point (1, 1, – 1) from the plane 3x + 4y - 12z

$$= 0 \text{ is } \left| \frac{3(1) + 4(1 - 12(-1))}{\sqrt{3^2 + 4^2 + (-12)^2}} \right|$$
$$= \left| \frac{3 + 4 + 12}{\sqrt{9 + 16 + 144}} \right|$$
$$= \frac{19}{\sqrt{169}}$$
$$= \frac{19}{13}$$

= 19units.

Miscellaneous Exercise 6 B | Q 19 | Page 226

Solve the following :

Find the vector equation of the plane passing through the origin and containing the line $\bar{\mathbf{r}} = (\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + \hat{\mathbf{k}}) + \lambda(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}})$.

Solution:

The vector equation of the plane passing through $A(\bar{a})$ and perpendicular to the vector \bar{n} is $\bar{r} \cdot \bar{n} = \bar{a} \cdot \bar{n}$...(1)

We can take $\bar{\mathbf{a}} = \bar{\mathbf{0}}$ since the plane passes through the origin.



The point M with position vector $\overline{m} = \hat{i} + 4\hat{j} + \hat{k}$ lies on the line and hence it lies on the plane.

 $\therefore \, \overline{OM} = \overline{m} = \hat{i} + 4 \hat{j} + \hat{k}$ lies on the plane.

The plane contains the given line which is parallel to $ar{b}=\hat{i}+2\hat{j}+\hat{k}.$

Let \bar{n} be normal to the plane. Then \bar{n} is perpendicular to $\overline{OM}\,$ as well as $\bar{b}\,$

$$\hat{\mathbf{n}} = \overline{\mathbf{OM}} \times \overline{\mathbf{b}} = \begin{vmatrix} \hat{\mathbf{i}} \\ 1 & 4 & 1 \\ 1 & 2 & 1 \end{vmatrix}$$
$$= (4-2)\hat{\mathbf{i}} - (1-1)\hat{\mathbf{j}} + (2-4)\hat{\mathbf{k}}$$
$$= 2\hat{\mathbf{i}} - 2\hat{\mathbf{k}}$$
$$\hat{\mathbf{n}} \text{ from (1), the vector equation of the required plane is }$$

$$\mathbf{\bar{r}} \cdot \left(2\hat{\mathbf{i}} - 2\hat{\mathbf{k}} \right) = \mathbf{\bar{0}} \cdot \mathbf{\bar{n}} = \mathbf{0}$$

i.e. $\mathbf{\bar{r}} \cdot \left(\hat{\mathbf{i}} - \hat{\mathbf{k}} \right) = \mathbf{0}$.

Miscellaneous Exercise 6 B | Q 20 | Page 226

Solve the following :

Find the vector equation of the plane which bisects the segment joining A(2, 3, 6) and B(4, 3, -2) at right angle.

Solution:

The vector equation of the plane passing through $A(\bar{a})$ and perpendicular to the vector \bar{n} is $\bar{r} \cdot \bar{n} = \bar{a} \cdot \bar{n}$...(1)



The position vectors $\bar{\mathbf{a}}$ and $\bar{\mathbf{b}}$ of the given points A and B are $\bar{\mathbf{a}} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$ and $\bar{\mathbf{b}} = 4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$

If M is the midpoint of segment AB, the position vector $\overline{\mathbf{m}}$ of M is given by

$$\begin{split} \overline{\mathbf{m}} &= \frac{\bar{\mathbf{a}} + \bar{\mathbf{b}}}{2} \\ &= \frac{\left(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 6\hat{\mathbf{k}}\right) + \left(4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}\right)}{2} \\ &= \frac{6\hat{\mathbf{i}} + 6\hat{\mathbf{j}} + 4\hat{\mathbf{k}}}{2} \\ &= 3\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 2\hat{\mathbf{k}} \end{split}$$

The plane passes through $M(\overline{m})$.

AB is perpendicular to the plane If $\mathbf{\bar{n}}$ is normal to the plane, then $\mathbf{\bar{n}} = \overline{AB}$ $\therefore \mathbf{\bar{n}} = \mathbf{\bar{b}} - \mathbf{\bar{a}} = (4\mathbf{\hat{i}} + 3\mathbf{\hat{j}} - 2\mathbf{\hat{k}}) - (2\mathbf{\hat{i}} + 3\mathbf{\hat{j}} + 6\mathbf{\hat{k}})$ $= 2\mathbf{\hat{i}} - 8\mathbf{\hat{k}}$ $\therefore \mathbf{\bar{m}} \cdot \mathbf{\bar{n}} = (3\mathbf{\hat{i}} + 3\mathbf{\hat{j}} + 2\mathbf{\hat{k}}) \cdot (2\mathbf{\hat{i}} - 8\mathbf{\hat{k}})$ = (3)(2) + (3)(0) + (2)(-8) = 6 + 0 - 16 = -10 \therefore from (1), the vector equation of the required plane is $\mathbf{\bar{r}} \cdot \mathbf{\bar{n}} = \mathbf{\bar{m}} \cdot \mathbf{\bar{n}}$



i.e.
$$\mathbf{\bar{r}} \cdot \left(2\mathbf{\hat{i}} - 8\mathbf{\hat{k}}\right) = -10$$

i.e. $\mathbf{\bar{r}} \cdot \left(\mathbf{\hat{i}} - 4\mathbf{\hat{k}}\right) = -5$.

Miscellaneous Exercise 6 B | Q 21 | Page 226

Solve the following :

Show that the lines x = y, z = 0 and x + y = 0, z = 0 intersect each other. Find the vector equation of the plane determined by them.

Solution:

Given lines are x = y, z = 0 and x + y = 0, z = 0.

It is clear that (0, 0, 0) satisfies both the equations.

 \therefore the lines intersect at O whose position vector is $ar{\mathbf{0}}$

Since z = 0 fr both the lines, both the lines ie in XY-plane.

Hence, we have to find equation oXY-ane.

Z-axis is perpendicular to XY-plane.

 \therefore normal to XY plane is $\hat{\mathbf{k}}$.

 $O(\overline{0})$ lies on the plane.

By using $\bar{\mathbf{r}}. \bar{\mathbf{n}} = \bar{\mathbf{a}}. \bar{\mathbf{n}}$, vecttor equation of the required plane is $\bar{\mathbf{r}}. \hat{\mathbf{k}} = \bar{\mathbf{0}}. \bar{\mathbf{k}}$

i.e. $\overline{\mathbf{r}} \cdot \hat{\mathbf{k}} = 0$.

Hence, the given lines intersect each other and the vector equation of the plane determine by them is $\mathbf{\bar{r}} \cdot \mathbf{\hat{k}} = 0$.