

Mathematical Logic

EXERCISE 1.1 [PAGES 6 - 8]

Exercise 1.1 | Q 1.01 | Page 6

State which of the following is the statement. Justify. In case of a statement, state its truth value.

$$5 + 4 = 13$$

Solution: It is a statement which is false, hence its truth value is 'F'.

Exercise 1.1 | Q 1.02 | Page 6

State which of the following is the statement. Justify. In case of a statement, state its truth value.

$$x - 3 = 14$$

Solution: It is an open sentence, hence it is not a statement.

Exercise 1.1 | Q 1.03 | Page 6

State which of the following is the statement. Justify. In case of a statement, state its truth value.

Close the door.

Solution: It is an imperative sentence, hence it is not a statement.

Exercise 1.1 | Q 1.04 | Page 6

State which of the following is the statement. Justify. In case of a statement, state its truth value.

Zero is a complex number.

Solution: It is a statement which is true, hence its truth value is 'T'.

Exercise 1.1 | Q 1.05 | Page 6

State which of the following is the statement. Justify. In case of a statement, state its truth value.

Please get me breakfast.

Solution: It is an imperative sentence, hence it is not a statement.

Exercise 1.1 | Q 1.06 | Page 6

State which of the following is the statement. Justify. In case of a statement, state its truth value.

Congruent triangles are similar.

Solution: It is a statement which is true, hence its truth value is 'T'.

Exercise 1.1 | Q 1.07 | Page 6

State which of the following is the statement. Justify. In case of a statement, state its truth value.

$$x^2 = x$$

Solution: It is an open sentence, hence it is not a statement.

Exercise 1.1 | Q 1.08 | Page 8

State which of the following is the statement. Justify. In case of a statement, state its truth value.

A quadratic equation cannot have more than two roots.

Solution: It is a statement which is true, hence its truth value is 'T'.

Exercise 1.1 | Q 1.09 | Page 7

State which of the following is the statement. Justify. In case of a statement, state its truth value.

Do you like Mathematics?

Solution: It is an interrogative sentence, hence it is not a statement.

Exercise 1.1 | Q 1.1 | Page 7

State which of the following is the statement. Justify. In case of a statement, state its truth value.

The sunsets in the west

Solution: It is a statement which is true, hence its truth value is 'T'.

Exercise 1.1 | Q 1.11 | Page 7

State which of the following is the statement. Justify. In case of a statement, state its truth value.

All real numbers are whole numbers.

Solution: It is a statement which is false, hence its truth value is 'F'.

Exercise 1.1 | Q 1.12 | Page 7

State which of the following is the statement. Justify. In case of a statement, state its truth value.

Can you speak in Marathi?

Solution: It is an interrogative sentence, hence it is not a statement.

Exercise 1.1 | Q 1.13 | Page 7

State which of the following is the statement. Justify. In case of a statement, state its truth value.

$$x^2 - 6x - 7 = 0, \text{ when } x = 7$$

Solution: It is a statement which is true, hence its truth value is 'T'.

Exercise 1.1 | Q 1.14 | Page 7

State which of the following is the statement. Justify. In case of a statement, state its truth value.

The sum of cube roots of unity is zero.

Solution: It is a statement which is true, hence its truth value is 'T'.

Exercise 1.1 | Q 1.15 | Page 7

State which of the following is the statement. Justify. In case of a statement, state its truth value.

It rains heavily.

Solution: It is an open sentence, hence it is not a statement.

Exercise 1.1 | Q 2.1 | Page 7

Write the following compound statement symbolically.

Nagpur is in Maharashtra and Chennai is in Tamil Nadu.

Solution: Let p: Nagpur is in Maharashtra.

Let q: Chennai is in Tamil Nadu.

Then the symbolic form of the given statement is $p \wedge q$.

Exercise 1.1 | Q 2.2 | Page 7

Write the following compound statement symbolically.

Triangle is equilateral or isosceles.

Solution: Let p: Triangle is equilateral.

Let q: Triangle is isosceles.

Then the symbolic form of the given statement is $p \vee q$.

Exercise 1.1 | Q 2.3 | Page 7

Write the following compound statement symbolically.

The angle is right angle if and only if it is of measure 90° .

Solution: Let p: The angle is right angle.

Let q: It is of measure 90°

Then the symbolic form of the given statement is $p \leftrightarrow q$.

Exercise 1.1 | Q 2.4 | Page 7

Write the following compound statement symbolically.

Angle is neither acute nor obtuse.

Solution: Let p: Angle is acute.

Let q: Angle is obtuse.

Then the symbolic form of the given statement is $\sim p \wedge \sim q$.

Exercise 1.1 | Q 2.5 | Page 7

Write the following compound statement symbolically.

If ΔABC is right-angled at B, then $m\angle A + m\angle C = 90^\circ$

Solution: Let p: ΔABC is right-angled at B.

Let q: $m\angle A + m\angle C = 90^\circ$

Then the symbolic form of the given statement is $p \rightarrow q$.

Exercise 1.1 | Q 2.6 | Page 7

Write the following compound statement symbolically.

Hima Das wins gold medal if and only if she runs fast.

Solution: Let p: Hima Das wins gold medal

Let q: She runs fast.

Then the symbolic form of the given statement is $p \leftrightarrow q$.

Exercise 1.1 | Q 2.7 | Page 7

Write the following compound statement symbolically.

x is not irrational number but is a square of an integer.

Solution: Let p: x is not irrational number

Let q: It is a square of an integer

Then the symbolic form of the given statement is $p \wedge q$.

[**Note:** If p: x is irrational number, then the symbolic form of the given statement is $\sim p \wedge q$.]

Exercise 1.1 | Q 3.1 | Page 7

Write the truth values of the following.

4 is odd or 1 is prime.

Solution: Let p: 4 is odd.

q: 1 is prime.

Then the symbolic form of the given statement is $p \vee q$.

The truth values of both p and q are F.

\therefore The truth value of $p \vee q$ is F[F \vee F \equiv F]

Exercise 1.1 | Q 3.2 | Page 7

Write the truth values of the following.

64 is a perfect square and 46 is a prime number.

Solution: Let p: 64 is a perfect square.

q: 46 is a prime number.

Then the symbolic form of the given statement is $p \wedge q$.

The truth values of p and q are T and F respectively. \therefore The truth value of $p \wedge q$ is F[T \wedge F \equiv F]

Exercise 1.1 | Q 3.3 | Page 7

Write the truth values of the following.

5 is a prime number and 7 divides 94.

Solution: Let p: 5 is a prime number.

q: 7 divides 94.

Then the symbolic form of the given statement is $p \wedge q$.

The truth values of p and q are T and F respectively.

\therefore The truth value of $p \wedge q$ is F[T \wedge F \equiv F]

Exercise 1.1 | Q 3.4 | Page 7

Write the truth values of the following.

It is not true that $5-3i$ is a real number.

Solution: Let p : $5-3i$ is a real number.

Then the symbolic form of the given statement is $\sim p$.

The truth values of p is F.

\therefore The truth values of $\sim p$ is T [$\sim F \equiv T$]

Exercise 1.1 | Q 3.5 | Page 7

Write the truth value of the following.

If $3 \times 5 = 8$ then $3 + 5 = 15$.

Solution: Let p : $3 \times 5 = 8$

q : $3 + 5 = 15$

Then the symbolic form of the given statement is $p \rightarrow q$.

The truth values of both p and q are F.

\therefore The truth value of $p \rightarrow q$ is T [$F \rightarrow F \equiv T$]

Exercise 1.1 | Q 3.6 | Page 7

Write the truth value of the following.

Milk is white if and only if sky is blue.

Solution: Let p : Milk is white.

q : Sky is blue

Then the symbolic form of the given statement is $p \leftrightarrow q$.

The truth values of both p and q are T.

\therefore The truth value of $p \leftrightarrow q$ is T [$T \leftrightarrow T \equiv T$]

Exercise 1.1 | Q 3.7 | Page 7

Write the truth values of the following.

24 is a composite number or 17 is a prime number.

Solution: Let p : 24 is a composite number.

q : 17 is a prime number.

Then the symbolic form of the given statement is $p \vee q$.

The truth values of both p and q are T.

∴ The truth value of $p \vee q$ is T[$T \vee T \equiv T$]

Exercise 1.1 | Q 4.1 | Page 7

If the statement p, q are true statement and r, s are false statement then determine the truth value of the following:

$$p \vee (q \wedge r)$$

Solution: Truth values of p and q are T and truth values of r and s are F.

$$p \vee (q \wedge r) \equiv T \vee (T \wedge F)$$

$$\equiv T \vee F \equiv T$$

Hence the truth value of the given statement is true.

Exercise 1.1 | Q 4.2 | Page 7

If the statement p, q are true statement and r, s are false statement then determine the truth value of the following:

$$(p \rightarrow q) \vee (r \rightarrow s)$$

Solution: Truth values of p and q are T and truth values of r and s are F.

$$(p \rightarrow q) \vee (r \rightarrow s) \equiv (T \rightarrow T) \vee (F \rightarrow F)$$

$$\equiv T \vee T \equiv T$$

Hence the truth value of the given statement is true.

Exercise 1.1 | Q 4.3 | Page 7

If the statement p, q are true statement and r, s are false statement then determine the truth value of the following:

$$(q \wedge r) \vee (\sim p \wedge s)$$

Solution: Truth values of p and q are T and truth values of r and s are F.

$$(q \wedge r) \vee (\sim p \wedge s) \equiv (T \wedge F) \vee (\sim T \wedge F)$$

$$\equiv F \vee (F \wedge F)$$

$$\equiv F \vee F \equiv F$$

Hence the truth value of the given statement is false.

Exercise 1.1 | Q 4.4 | Page 7

If the statement p, q are true statement and r, s are false statement then determine the truth value of the following:

$$(p \rightarrow q) \wedge \sim r$$

Solution: Truth values of p and q are T and truth values of r and s are F.

$$(p \rightarrow q) \wedge (\sim r) \equiv (T \rightarrow T) \wedge (\sim F) \\ \equiv T \wedge T \equiv T$$

Hence the truth value of the given statement is true.

Exercise 1.1 | Q 4.5 | Page 7

If the statement p, q are true statement and r, s are false statement then determine the truth value of the following:

$$(\sim r \leftrightarrow p) \rightarrow \sim q$$

Solution: Truth values of p and q are T and truth values of r and s are F.

$$(\sim r \leftrightarrow p) \rightarrow (\sim q) \equiv (\sim F \leftrightarrow T) \rightarrow (\sim T) \\ \equiv (T \leftrightarrow T) \rightarrow F \\ \equiv T \rightarrow F \equiv F$$

Hence the truth value of the given statement is false.

Exercise 1.1 | Q 4.6 | Page 7

If the statement p, q are true statement and r, s are false statement then determine the truth value of the following:

$$[\sim p \wedge (\sim q \wedge r)] \vee [(q \wedge r) \vee (p \wedge r)]$$

Solution: Truth values of p and q are T and truth values of r and s are F.

$$[\sim p \wedge (\sim q \wedge r)] \vee [(q \wedge r) \vee (p \wedge r)] \\ \equiv [\sim T \wedge (\sim T \wedge F)] \vee [(T \wedge F) \vee (T \wedge F)] \\ \equiv [F \wedge (F \wedge F)] \vee [F \vee F] \\ \equiv (F \wedge F) \vee F \\ \equiv F \vee F \equiv F$$

Hence the truth value of the given statement is false.

Exercise 1.1 | Q 4.7 | Page 7

If the statement p, q are true statement and r, s are false statement then determine the truth value of the following:

$$[(\sim p \wedge q) \wedge \sim r] \vee [(q \rightarrow p) \rightarrow (\sim s \vee r)]$$

Solution: Truth values of p and q are T and truth values of r and s are F.

$$[(\sim p \wedge q) \wedge (\sim r)] \vee [(q \rightarrow p) \rightarrow (\sim s \vee r)] \\ \equiv [(\sim T \wedge T) \wedge (\sim F)] \vee [(T \rightarrow T) \rightarrow (\sim F \vee F)] \\ \equiv [(F \wedge T) \wedge T] \vee [T \rightarrow (T \vee F)] \\ \equiv (F \wedge T) \vee (T \rightarrow T) \\ \equiv F \vee T \equiv T$$

Hence the truth value of the given statement is true.

Exercise 1.1 | Q 4.8 | Page 7

If the statement p, q are true statement and r, s are false statement then determine the truth value of the following:

$$\sim [(\sim p \wedge r) \vee (s \rightarrow \sim q)] \leftrightarrow (p \wedge r)$$

Solution: Truth values of p and q are T and truth values of r and s are F.

$$\begin{aligned} &\sim [(\sim p \wedge r) \vee (s \rightarrow \sim q)] \leftrightarrow (p \wedge r) \\ &\equiv \sim [(\sim T \wedge F) \vee (F \rightarrow \sim T)] \leftrightarrow (T \wedge F) \\ &\equiv \sim [(F \wedge F) \vee (F \rightarrow F)] \leftrightarrow F \\ &\equiv \sim (F \vee T) \leftrightarrow F \\ &\equiv \sim T \leftrightarrow F \\ &\equiv F \leftrightarrow F \equiv T \end{aligned}$$

Hence the truth value of the given statement is true.

Exercise 1.1 | Q 5.1 | Page 7

Write the negation of the following.

Tirupati is in Andhra Pradesh.

Solution: The negation of the given statement is:

Tirupati is not in Andhra Pradesh.

Exercise 1.1 | Q 5.2 | Page 7

Write the negation of the following.

3 is not a root of the equation $x^2 + 3x - 18 = 0$

Solution: The negation of the given statement is:

3 is a root of the equation $x^2 + 3x - 18 = 0$

Exercise 1.1 | Q 5.3 | Page 7

Write the negation of the following.

$\sqrt{2}$ is a rational number.

Solution: The negation of the given statement is:

$\sqrt{2}$ is not a rational number.

Exercise 1.1 | Q 5.4 | Page 7

Write the negation of the following.

Polygon ABCDE is a pentagon.

Solution: The negation of the given statement is:

Polygon ABCDE is not a pentagon.

Exercise 1.1 | Q 5.5 | Page 7

Write the negation of the following.

$$7 + 3 > 5$$

Solution: $7 + 3 \ngtr 5$

EXERCISE 1.2 [PAGE 13]

Exercise 1.2 | Q 1.01 | Page 13

Construct the truth table of the following statement pattern.

$$[(p \rightarrow q) \wedge q] \rightarrow p$$

Solution: Here are two statements and three connectives.

\therefore There are $2 \times 2 = 4$ rows and $2 + 3 = 5$ columns in the truth table.

p	q	$p \rightarrow q$	$(p \rightarrow q) \wedge q$	$[(p \rightarrow q) \wedge q] \rightarrow p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

Exercise 1.2 | Q 1.02 | Page 13

Construct the truth table of the following statement pattern.

$$(p \wedge \sim q) \leftrightarrow (p \rightarrow q)$$

Solution:

p	q	$\sim q$	$p \wedge \sim q$	$p \rightarrow q$	$(p \wedge \sim q) \leftrightarrow (p \rightarrow q)$
T	T	F	F	T	F
T	F	T	T	F	F
F	T	F	F	T	F
F	F	T	F	T	F

Exercise 1.2 | Q 1.03 | Page 13

Construct the truth table of the following statement pattern.

$$(p \wedge q) \leftrightarrow (q \vee r)$$

Solution:

p	q	r	$p \wedge q$	$q \vee r$	$(p \wedge q) \leftrightarrow (q \vee r)$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	T	F
T	F	F	F	F	T
F	T	T	F	T	F
F	T	F	F	T	F
F	F	T	F	T	F
F	F	F	F	F	T

Exercise 1.2 | Q 1.04 | Page 13

Construct the truth table of the following statement pattern.

$$p \rightarrow [\sim (q \wedge r)]$$

Solution:

p	q	r	$q \wedge r$	$\sim (q \wedge r)$	$p \rightarrow [\sim (q \wedge r)]$
T	T	T	T	F	F
T	T	F	F	T	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	F	T
F	T	F	F	T	T
F	F	T	F	T	T
F	F	F	F	T	T

Exercise 1.2 | Q 1.05 | Page 13

Construct the truth table of the following statement pattern.

$$\sim p \wedge [(p \vee \sim q) \wedge q]$$

Solution:

p	q	$\sim p$	$\sim q$	$p \vee \sim q$	$(p \vee \sim q) \wedge q$	$\sim p \wedge [p \vee \sim q] \wedge q$
T	T	F	F	T	T	F
T	F	F	T	T	F	F
F	T	T	F	F	F	F
F	F	T	T	T	F	F

Exercise 1.2 | Q 1.06 | Page 13

Construct the truth table of the following statement pattern.

$$(\sim p \rightarrow \sim q) \wedge (\sim q \rightarrow \sim p)$$

Solution:

p	q	$\sim p$	$\sim q$	$\sim p \rightarrow \sim q$	$\sim q \rightarrow \sim p$	$(\sim p \rightarrow \sim q) \wedge (\sim q \rightarrow \sim p)$
T	T	F	F	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	T	T	T

Exercise 1.2 | Q 1.07 | Page 13

Construct the truth table of the following statement pattern.

$$(q \rightarrow p) \vee (\sim p \leftrightarrow q)$$

Solution:

p	q	$\sim p$	$q \rightarrow p$	$\sim p \leftrightarrow q$	$(q \rightarrow p) \vee (\sim p \leftrightarrow q)$
T	T	F	T	F	T
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	F	T

Exercise 1.2 | Q 1.08 | Page 13

Construct the truth table of the following statement pattern.

$$[p \rightarrow (q \rightarrow r)] \leftrightarrow [(p \wedge q) \rightarrow r]$$

Solution:

p	q	r	$q \rightarrow r$	$p \rightarrow (q \rightarrow r)$	$p \wedge q$	$(p \wedge q) \rightarrow r$	$[p \rightarrow (q \rightarrow r)] \leftrightarrow [(p \wedge q) \rightarrow r]$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	T
T	F	T	T	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	T	T	F	T	T
F	T	F	F	T	F	T	T
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

Exercise 1.2 | Q 1.09 | Page 13

Construct the truth table of the following statement pattern.

$$p \rightarrow [\sim (q \wedge r)]$$

Solution:

p	q	r	$q \wedge r$	$\sim (q \wedge r)$	$p \rightarrow [\sim (q \wedge r)]$
T	T	T	T	F	F
T	T	F	F	T	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	F	T
F	T	F	F	T	T
F	F	T	F	T	T
F	F	F	F	T	T

Exercise 1.2 | Q 1.1 | Page 13

Construct the truth table of the following statement pattern.

$$(p \vee \sim q) \rightarrow (r \wedge p)$$

Solution:

p	q	r	$\sim q$	$p \vee \sim q$	$r \wedge p$	$(p \vee \sim q) \rightarrow (r \wedge p)$
T	T	T	F	T	T	T
T	T	F	F	T	F	F
T	F	T	T	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	F	T
F	T	F	F	F	F	T
F	F	T	T	T	F	F
F	F	F	T	T	F	F

Exercise 1.2 | Q 2.01 | Page 13

Using truth table, prove that $\sim p \wedge q \equiv (p \vee q) \wedge \sim p$

Solution:

1	2	3	4	5	6
p	q	$\sim p$	$\sim p \wedge q$	$p \vee q$	$(p \vee q) \wedge \sim p$
T	T	F	F	T	F
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	F	F	F

The entries in columns 4 and 6 are identical

$$\therefore \sim p \wedge q \equiv (p \vee q) \wedge \sim p$$

Exercise 1.2 | Q 2.02 | Page 13

Using the truth table prove the following logical equivalence.

$$\sim (p \vee q) \vee (\sim p \wedge q) \equiv \sim p$$

Solution:

1	2	3	4	5	6	7
p	q	$\sim p$	$p \vee q$	$\sim (p \vee q)$	$\sim p \wedge q$	$\sim (p \vee q) \vee (\sim p \wedge q)$
T	T	F	T	F	F	F
T	F	F	T	F	F	F
F	T	T	T	F	T	T
F	F	T	F	T	F	T

The entries in columns 3 and 7 are identical.

$$\therefore \sim (p \vee q) \vee (\sim p \wedge q) \equiv \sim p$$

Exercise 1.2 | Q 2.03 | Page 13

Using the truth table prove the following logical equivalence.

$$p \leftrightarrow q \equiv \sim [(p \vee q) \wedge \sim (p \wedge q)]$$

Solution:

1	2	3	4	5	6	7	8
p	q	$p \leftrightarrow q$	$p \vee q$	$p \wedge q$	$\sim (p \wedge q)$	$(p \vee q) \wedge \sim (p \wedge q)$	$\sim [(p \vee q) \wedge \sim (p \wedge q)]$
T	T	T	T	T	F	F	T
T	F	F	T	F	T	T	F
F	T	F	T	F	T	T	F
F	F	T	F	F	T	F	T

The entries in columns 3 and 8 are identical.

$$\therefore p \leftrightarrow q \equiv \sim [(p \vee q) \wedge \sim (p \wedge q)]$$

Exercise 1.2 | Q 2.04 | Page 13

Using the truth table prove the following logical equivalence.

$$p \rightarrow (q \rightarrow p) \equiv \sim p \rightarrow (p \rightarrow q)$$

Solution:

1	2	3	4	5	6	7
p	q	$q \rightarrow p$	$p \rightarrow (q \rightarrow p)$	$\sim p$	$p \rightarrow q$	$\sim p \rightarrow (p \rightarrow q)$
T	T	T	T	F	T	T
T	F	T	T	F	F	T
F	T	F	T	T	T	T
F	F	T	T	T	T	T

The entries in columns 4 and 7 are identical.

$$\therefore p \rightarrow (q \rightarrow p) \equiv \sim p \rightarrow (p \rightarrow q)$$

Exercise 1.2 | Q 2.05 | Page 13

Using the truth table prove the following logical equivalence.

$$(p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$$

Solution:

1	2	3	4	5	6	7	8
p	q	r	$p \vee q$	$(p \vee q) \rightarrow r$	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r) \wedge (q \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	F	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

The entries in columns 5 and 8 are identical.

$$\therefore (p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$$

Exercise 1.2 | Q 2.06 | Page 13

Using the truth table prove the following logical equivalence.

$$p \rightarrow (q \wedge r) \equiv (p \rightarrow q) \wedge (p \rightarrow r)$$

Solution:

1	2	3	4	5	6	7	8
p	q	r	$q \wedge r$	$p \rightarrow (q \wedge r)$	$p \rightarrow q$	$p \rightarrow r$	$(p \rightarrow q) \wedge (p \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F
T	F	T	F	F	F	T	F
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

The entries in columns 5 and 8 are identical.

$$\therefore p \rightarrow (q \wedge r) \equiv (p \rightarrow q) \wedge (p \rightarrow r)$$

Exercise 1.2 | Q 2.07 | Page 13

Using the truth table prove the following logical equivalence.

$$p \rightarrow (q \wedge r) \equiv (p \wedge q) (p \rightarrow r)$$

Solution:

1	2	3	4	5	6	7	8
p	q	r	$q \wedge r$	$p \rightarrow (q \wedge r)$	$(p \wedge q)$	$(p \rightarrow r)$	$(p \wedge q) (p \rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F
T	F	T	F	F	F	T	F
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

The entries in columns 5 and 8 are identical.

$$\therefore p \rightarrow (q \wedge r) \equiv (p \wedge q) (p \rightarrow r)$$

Exercise 1.2 | Q 2.08 | Page 13

Using the truth table prove the following logical equivalence.

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

Solution:

1	2	3	4	5	6	7	8
p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$p \wedge q$	$p \wedge r$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

The entries in columns 5 and 8 are identical.

$$\therefore p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

Exercise 1.2 | Q 2.09 | Page 13

Using the truth table prove the following logical equivalence.

$$[\sim (p \vee q) \vee (p \vee q)] \wedge r \equiv r$$

Solution:

1	2	3	4	5	6	7
p	q	r	$p \vee r$	$\sim (p \vee q)$	$\sim (p \vee q) \vee (p \vee q)$	$[\sim (p \vee q) \vee (p \vee q)] \wedge r$
T	T	T	T	F	T	T
T	T	F	T	F	T	F
T	F	T	T	F	T	T
T	F	F	T	F	T	F
F	T	T	T	F	T	T
F	T	F	T	F	T	F

F	F	T	F	T	T	T
F	F	F	F	T	T	F

The entries in columns 3 and 7 are identical.

$$\therefore [\sim (p \vee q) \vee (p \vee q)] \wedge r \equiv r$$

Exercise 1.2 | Q 2.1 | Page 13

Using the truth table proves the following logical equivalence.

$$\sim (p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$$

Solution:

1	2	3	4	5	6	7	8	9
p	q	$\sim p$	$\sim q$	$p \leftrightarrow q$	$\sim (p \leftrightarrow q)$	$p \wedge \sim q$	$q \wedge \sim p$	$(p \wedge \sim q) \vee (q \wedge \sim p)$
T	T	F	F	T	F	F	F	F
T	F	F	T	F	T	T	F	T
F	T	T	F	F	T	F	T	T
F	F	T	T	T	F	F	F	F

The entries in columns 6 and 9 are identical.

$$\therefore \sim (p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$$

Exercise 1.2 | Q 3.01 | Page 13

Examine whether the following statement pattern is a tautology or a contradiction or a contingency.

$$(p \wedge q) \rightarrow (q \vee p)$$

Solution:

p	q	$p \wedge q$	$q \vee p$	$(p \wedge q) \rightarrow (q \vee p)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

All the entries in the last column of the above truth table are T.

$$\therefore (p \wedge q) \rightarrow (q \vee p) \text{ is a tautology.}$$

Exercise 1.2 | Q 3.02 | Page 13

Examine whether the following statement pattern is a tautology or a contradiction or a contingency.

$$(p \rightarrow q) \leftrightarrow (\sim p \vee q)$$

Solution:

p	q	$\sim p$	$p \rightarrow q$	$\sim p \vee q$	$(p \rightarrow q) \leftrightarrow (\sim p \vee q)$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

All the entries in the last column of the above truth table are T.

$\therefore (p \rightarrow q) \leftrightarrow (\sim p \vee q)$ is a tautology.

Exercise 1.2 | Q 3.03 | Page 13

Discuss the statement pattern, using truth table : $\sim(\sim p \wedge \sim q) \vee q$

Solution: Consider the statement pattern: $\sim(\sim p \wedge \sim q) \vee q$

Thus the truth table of the given logical statement: $\sim(\sim p \wedge \sim q) \vee q$

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$\sim(\sim p \wedge \sim q)$	$\sim(\sim p \wedge \sim q) \vee q$
T	T	F	F	F	T	T
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	T	F	F

The above statement is **contingency**.

Exercise 1.2 | Q 3.04 | Page 13

Examine whether the following logical statement pattern is a tautology, contradiction, or contingency.

$$[(p \rightarrow q) \wedge q] \rightarrow p$$

Solution: Consider the statement pattern : $[(p \rightarrow q) \wedge q] \rightarrow p$

No. of rows = $2^n = 2 \times 2 = 4$

No. of column = $m + n = 3 + 2 = 5$

Thus the truth table of the given logical statement :

$$[(p \rightarrow q) \wedge q] \rightarrow p$$

p	q	$p \rightarrow q$	$(p \rightarrow q) \wedge q$	$[(p \rightarrow q) \wedge q] \rightarrow p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

From the above truth table we can say that given logical statement: $[(p \rightarrow q) \wedge q] \rightarrow p$ is contingency.

Exercise 1.2 | Q 3.05 | Page 13

Examine whether the following statement pattern is a tautology or a contradiction or a contingency.

$$[(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$$

Solution:

p	q	$\sim p$	$\sim q$	$p \rightarrow q$	$(p \rightarrow q) \wedge \sim q$	$[(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

All the entries in the last column of the above truth table are T.

$\therefore [(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$ is a tautology.

Exercise 1.2 | Q 3.06 | Page 13

Examine whether the following statement pattern is a tautology or a contradiction or a contingency.

$$(p \leftrightarrow q) \wedge (p \rightarrow \sim q)$$

Solution:

p	q	$\sim q$	$p \leftrightarrow q$	$p \rightarrow \sim q$	$(p \leftrightarrow q) \wedge (p \rightarrow \sim q)$
T	T	F	T	F	F
T	F	T	F	T	F
F	T	F	F	T	F
F	F	T	T	T	T

The entries in the last column of the above truth table are neither all T nor all F.
 $\therefore (p \leftrightarrow q) \wedge (p \rightarrow \sim q)$ is a contingency.

Exercise 1.2 | Q 3.07 | Page 13

Examine whether the following statement pattern is a tautology or a contradiction or a contingency.

$$\sim (\sim q \wedge p) \wedge q$$

Solution:

p	q	$\sim q$	$\sim q \wedge p$	$\sim (\sim q \wedge p)$	$\sim (\sim q \wedge p) \wedge q$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	F	T	T
F	F	T	F	T	F

The entries in the last column of the above truth table are neither all T nor all F.
 $\therefore \sim (\sim q \wedge p) \wedge q$ is a contingency.

Exercise 1.2 | Q 3.08 | Page 13

Examine whether the following statement pattern is a tautology or a contradiction or a contingency.

$$(p \wedge \sim q) \leftrightarrow (p \rightarrow q)$$

Solution:

p	q	$\sim q$	$p \wedge \sim q$	$p \rightarrow q$	$(p \wedge \sim q) \leftrightarrow (p \rightarrow q)$
T	T	F	F	T	F
T	F	T	T	F	F

F	T	F	F	T	F
F	F	T	F	T	F

All the entries in the last column of the above truth table are F.

$\therefore (p \wedge \sim q) \leftrightarrow (p \rightarrow q)$ is a contradiction.

[Note: Answer in the textbook is incorrect]

Exercise 1.2 | Q 3.09 | Page 13

Examine whether the following statement pattern is a tautology or a contradiction or a contingency.

$$(\sim p \rightarrow q) \wedge (p \wedge r)$$

Solution:

p	q	r	$\sim p$	$\sim p \rightarrow q$	$p \wedge r$	$(\sim p \rightarrow q) \wedge (p \wedge r)$
T	T	T	F	T	T	T
T	T	F	F	T	F	F
T	F	T	F	T	T	T
T	F	F	F	T	F	F
F	T	T	T	T	F	F
F	T	F	T	T	F	F
F	F	T	T	F	F	F
F	F	F	T	F	F	F

The entries in the last column of the above truth table are neither all T nor all F.

$\therefore (\sim p \rightarrow q) \wedge (p \wedge r)$ is a contingency.

Exercise 1.2 | Q 3.1 | Page 13

Examine whether the following statement pattern is a tautology or a contradiction or a contingency.

$$[p \rightarrow (\sim q \vee r)] \leftrightarrow \sim [p \rightarrow (q \rightarrow r)]$$

Solution:

p	q	r	$\sim q$	$\sim q \vee r$	$p \rightarrow (\sim q \vee r)$	$q \rightarrow r$	$p \rightarrow (q \rightarrow r)$	$\sim [p \rightarrow (q \rightarrow r)]$	$[p \rightarrow (\sim q \vee r)] \leftrightarrow [p \rightarrow (q \rightarrow r)]$
T	T	T	F	T	T	T	T	F	F
T	T	F	F	F	F	F	F	T	F

T	F	T	T	T	T	T	T	F	F
T	F	F	T	T	T	T	T	F	F
F	T	T	F	T	T	T	T	F	F
F	T	F	F	F	T	F	T	F	F
F	F	T	T	T	T	T	T	F	F
F	F	F	T	T	T	T	T	F	F

All the entries in the last column of the above truth table are F.

$\therefore [p \rightarrow (\sim q \vee r)] \leftrightarrow \sim [p \rightarrow (q \rightarrow r)]$ is a contradiction.

EXERCISE 1.3 [PAGES 17 - 18]

Exercise 1.3 | Q 1.1 | Page 17

If $A = \{3, 5, 7, 9, 11, 12\}$, determine the truth value of the following.

$\exists x \in A$ such that $x - 8 = 1$

Solution: Clearly $x = 9 \in A$ satisfies $x - 8 = 1$. So the given statement is true, hence its truth value is T.

Exercise 1.3 | Q 1.2 | Page 17

If $A = \{3, 5, 7, 9, 11, 12\}$, determine the truth value of the following.

$\forall x \in A$, $x^2 + x$ is an even number

Solution: For each $x \in A$, $x^2 + x$ is an even number. So the given statement is true, hence its truth value is T.

Exercise 1.3 | Q 1.3 | Page 17

If $A = \{3, 5, 7, 9, 11, 12\}$, determine the truth value of the following.

$\exists x \in A$ such that $x^2 < 0$

Solution: There is no $x \in A$ which satisfies $x^2 < 0$. So the given statement is false, hence its truth value is F.

Exercise 1.3 | Q 1.4 | Page 17

If $A = \{3, 5, 7, 9, 11, 12\}$, determine the truth value of the following.

$\forall x \in A$, x is an even number

Solution: $x = 3 \in A$, $x = 5 \in A$, $x = 7 \in A$, $x = 9 \in A$, $x = 11 \in A$ do not satisfy x is an even number. So the given statement is false, hence its truth value is F.

Exercise 1.3 | Q 1.5 | Page 17

If $A = \{3, 5, 7, 9, 11, 12\}$, determine the truth value of the following.

$$\exists x \in A \text{ such that } 3x + 8 > 40$$

Solution: Clearly $x = 11 \in A$ and $x = 12 \in A$ satisfies $3x + 8 > 40$. So the given statement is true, hence its truth value is T.

Exercise 1.3 | Q 1.6 | Page 17

If $A = \{3, 5, 7, 9, 11, 12\}$, determine the truth value of the following.

$$\forall x \in A, 2x + 9 > 14$$

Solution: For each $x \in A$, $2x + 9 > 14$. So the given statement is true, hence its truth value is T.

Exercise 1.3 | Q 2.01 | Page 17

Write the dual of the following.

$$p \vee (q \wedge r)$$

Solution: The dual of the given statement pattern is:

$$p \wedge (q \vee r)$$

Exercise 1.3 | Q 2.02 | Page 17

Write the dual of the following.

$$p \wedge (q \wedge r)$$

Solution: The dual of the given statement pattern is:

$$p \vee (q \vee r)$$

Exercise 1.3 | Q 2.03 | Page 17

Write the dual of the following.

$$(p \vee q) \wedge (r \vee s)$$

Solution: The dual of the given statement pattern is:

$$(p \wedge q) \vee (r \wedge s)$$

Exercise 1.3 | Q 2.04 | Page 17

Write the dual of the following.

$$p \wedge \sim q$$

Solution: The dual of the given statement pattern is:

$$p \vee \sim q$$

Exercise 1.3 | Q 2.05 | Page 17

Write the dual of the following.

$$(\sim p \vee q) \wedge (\sim r \wedge s)$$

Solution: The dual of the given statement pattern is:

$$(\sim p \wedge q) \vee (\sim r \vee s)$$

Exercise 1.3 | Q 2.06 | Page 17

Write the dual of the following.

$$\sim p \wedge (\sim q \wedge (p \vee q) \wedge \sim r)$$

Solution: The dual of the given statement pattern is:

$$\sim p \vee (\sim q \vee (p \wedge q) \vee \sim r)$$

Exercise 1.3 | Q 2.07 | Page 17

Write the dual of the following.

$$[\sim (p \vee q)] \wedge [p \vee \sim (q \wedge \sim s)]$$

Solution: The dual of the given statement pattern is:

$$[\sim (p \wedge q)] \vee [p \wedge \sim (q \vee \sim s)]$$

Exercise 1.3 | Q 2.08 | Page 17

Write the dual of the following.

$$c \vee \{p \wedge (q \vee r)\}$$

Solution: The dual of the given statement pattern is:

$$t \wedge \{p \vee (q \wedge r)\}$$

Exercise 1.3 | Q 2.09 | Page 17

Write the dual of the following.

$$\sim p \vee (q \wedge r) \wedge t$$

Solution: The dual of the given statement pattern is:

$$\sim p \wedge (q \vee r) \vee c$$

Exercise 1.3 | Q 2.1 | Page 17

Write the dual of the following.

$$(p \vee q) \vee c$$

Solution: The dual of the given statement pattern is:

$$(p \wedge q) \wedge t$$

Exercise 1.3 | Q 3.1 | Page 18

Write the negation of the following.

$$x + 8 > 11 \text{ or } y - 3 = 6$$

Solution: Let $p: x + 8 > 11$,

$$q: y - 3 = 6$$

Then the symbolic form of the given statement is $p \vee q$.

Since $\sim (p \vee q) \equiv \sim p \wedge \sim q$, the negation of the given statement is:

$$x + 8 \not> 11 \text{ and } y - 3 \neq 6.$$

OR

$$x + 8 \leq 11 \text{ and } y - 3 \neq 6.$$

Exercise 1.3 | Q 3.2 | Page 18

Write the negation of the following.

$$11 < 15 \text{ and } 25 > 20$$

Solution: Let $p: 11 < 15$,

$$q: 25 > 20$$

Then the symbolic form of the given statement is $p \wedge q$.

Since $\sim (p \wedge q) \equiv \sim p \vee \sim q$, the negation of the given statement is:

$$11 \geq 15 \text{ or } 25 \leq 20$$

Exercise 1.3 | Q 3.3 | Page 18

Write the negation of the following.

Quadrilateral is a square if and only if it is a rhombus.

Solution: Let p : Quadrilateral is a square.

q : It is a rhombus.

Then the symbolic form of the given statement is $p \leftrightarrow q$.

Since $\sim (p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$, the negation of the given statement is:
Quadrilateral is a square but it is not a rhombus or quadrilateral is a rhombus but it is not a square.

Exercise 1.3 | Q 3.4 | Page 18

Write the negation of the following.

It is cold and raining.

Solution: Let p: It is cold.

q: It is raining.

Then the symbolic form of the given statement is $p \wedge q$.

Since $\sim (p \wedge q) \equiv \sim p \vee \sim q$, the negation of the given statement is:

It is not cold or not raining.

Exercise 1.3 | Q 3.5 | Page 18

Write the negation of the following.

If it is raining then we will go and play football.

Solution: Let p: It is raining.

q: We will go.

r: We play football.

Then the symbolic form of the given statement is $p \rightarrow (q \wedge r)$.

Since $\sim [p \rightarrow (q \wedge r)] \equiv p \wedge \sim (q \wedge r) \equiv p \wedge (\sim q \vee \sim r)$, the negation of the given statement is:

It is raining and we will not go or not play football.

Exercise 1.3 | Q 3.6 | Page 18

Write the negation of the following.

$\sqrt{2}$ is a rational number.

Solution: The negation of the given statement is:

$\sqrt{2}$ is not a rational number.

Exercise 1.3 | Q 3.7 | Page 18

Write the negation of the following.

All-natural numbers are whole numbers.

Solution: The negation of the given statement is:

Some natural numbers are not whole numbers.

Exercise 1.3 | Q 3.8 | Page 18

Write the negation of the following.

$\forall n \in \mathbb{N}$, $n^2 + n + 2$ is divisible by 4.

Solution: The negation of the given statement is:

$\exists n \in \mathbb{N}$, such that $n^2 + n + 2$ is not divisible by 4.

Exercise 1.3 | Q 3.9 | Page 18

Write the negation of the following.

$\exists x \in \mathbb{N}$ such that $x - 17 < 20$

Solution: The negation of the given statement is:

$\forall x \in \mathbb{N}$, $x - 17 \geq 20$

Exercise 1.3 | Q 4.1 | Page 18

Write converse, inverse and contrapositive of the following statement.

If $x < y$ then $x^2 < y^2$ ($x, y \in \mathbb{R}$)

Solution: Let $p: x < y$,

$q: x^2 < y^2$

Then the symbolic form of the given statement is $p \rightarrow q$.

Converse: $q \rightarrow p$ is the converse of $p \rightarrow q$.

i.e. If $x^2 < y^2$, then $x < y$.

Inverse: $\sim p \rightarrow \sim q$ is the inverse of $p \rightarrow q$.

i.e. If $x \geq y$, then $x^2 \geq y^2$.

OR

If $x \nless y$, then $x^2 \nless y^2$.

Contrapositive: $\sim q \rightarrow \sim p$ is the contrapositive of $p \rightarrow q$

i.e. If $x^2 \geq y^2$, then $x \geq y$.

OR

If $x^2 \nless y^2$, then $x \nless y$.

Exercise 1.3 | Q 4.2 | Page 18

Write converse, inverse and contrapositive of the following statement.

A family becomes literate if the woman in it is literate.

Solution: Let p : The woman in the family is literate.

q : A family becomes literate.

Then the symbolic form of the given statement is $p \rightarrow q$.

Converse: $q \rightarrow p$ is the converse of $p \rightarrow q$.

i.e. If a family becomes literate, then the woman in it is literate.

Inverse: $\sim p \rightarrow \sim q$ is the inverse of $p \rightarrow q$.

i.e. If the woman in the family is not literate, then the family does not become literate.

Contrapositive: $\sim q \rightarrow \sim p$ is the contrapositive of $p \rightarrow q$.

i.e. If a family does not become literate, then the woman in it is not literate.

Exercise 1.3 | Q 4.3 | Page 18

Write converse, inverse and contrapositive of the following statement.

If surface area decreases then pressure increases.

Solution: Let p : The surface area decreases.

q : The pressure increases.

Then the symbolic form of the given statement is $p \rightarrow q$.

Converse: $q \rightarrow p$ is the converse of $p \rightarrow q$.

i.e. If the pressure increases, then the surface area decreases.

Inverse: $\sim p \rightarrow \sim q$ is the inverse of $p \rightarrow q$.

i.e. If the surface area does not decrease, then the pressure does not increase.

Contrapositive: $\sim q \rightarrow \sim p$ is the contrapositive of $p \rightarrow q$.

i.e. If the pressure does not increase, then the surface area does not decrease.

Exercise 1.3 | Q 4.4 | Page 18

Write converse, inverse and contrapositive of the following statement.

If voltage increases then current decreases.

Solution: Let p : Voltage increases.

q : Current decreases.

Then the symbolic form of the given statement is $p \rightarrow q$.

Converse: $q \rightarrow p$ is the converse of $p \rightarrow q$.

i.e. If current decreases, then voltage increases. **Inverse:** $\sim p \rightarrow \sim q$ is the inverse of $p \rightarrow q$.

i.e. If voltage does not increase, then-current does not decrease.

Contrapositive: $\sim q \rightarrow p$, is the contrapositive of $p \rightarrow q$.

i.e. If current does not decrease, then voltage does not increase.

EXERCISE 1.4 [PAGE 21]

Exercise 1.4 | Q 1.1 | Page 21

Using the rule of negation write the negation of the following with justification.

$$\sim q \rightarrow p$$

Solution: The negation of $\sim q \rightarrow p$ is

$$\sim (\sim q \rightarrow p) \equiv \sim q \wedge \sim p \dots \text{(Negation of implication)}$$

Exercise 1.4 | Q 1.2 | Page 21

Using the rule of negation write the negation of the following with justification.

$$p \wedge \sim q$$

Solution: The negation of $p \wedge \sim q$ is

$$\sim (p \wedge \sim q) \equiv \sim p \vee \sim (\sim q) \dots \text{(Negation of conjunction)}$$

$$\equiv \sim p \vee q \dots \text{(Negation of negation)}$$

Exercise 1.4 | Q 1.3 | Page 21

Using the rule of negation write the negation of the following with justification.

$$p \vee \sim q$$

Solution: The negation of $p \vee \sim q$ is

$$\sim (p \vee \sim q) \equiv \sim p \wedge \sim (\sim q) \dots \text{(Negation of disjunction)}$$

$$\equiv \sim p \wedge q \dots \text{(Negation of negation)}$$

Exercise 1.4 | Q 1.4 | Page 21

Using the rule of negation write the negation of the following with justification.

$$(p \vee \sim q) \wedge r$$

Solution: The negation of $(p \vee \sim q) \wedge r$ is

$$\sim [(p \vee \sim q) \wedge r] \equiv (p \vee \sim q) \vee \sim r \dots\dots\dots(\text{Negation of conjunction})$$

$$\equiv [\sim p \wedge \sim (\sim q)] \vee \sim r \dots\dots(\text{Negation of disjunction}) \equiv (\sim p \wedge q) \vee \sim r \dots\dots\dots(\text{Negation of negation})$$

Exercise 1.4 | Q 1.5 | Page 21

Using the rule of negation write the negation of the following with justification.

$$p \rightarrow (p \vee \sim q)$$

Solution: The negation of $p \rightarrow (p \vee \sim q)$ is

$$\sim [p \rightarrow (p \vee \sim q)] \equiv p \wedge \sim (p \vee \sim q) \dots\dots\dots(\text{Negation of implication})$$

$$\equiv p \wedge [\sim p \wedge \sim (\sim q)] \dots\dots(\text{Negation of disjunction})$$

$$\equiv p \wedge (\sim p \wedge q) \dots\dots\dots(\text{Negation of negation})$$

Exercise 1.4 | Q 1.6 | Page 21

Using the rule of negation write the negation of the following with justification.

$$\sim (p \wedge q) \vee (p \vee \sim q)$$

Solution: The negation of $\sim (p \wedge q) \vee (p \vee \sim q)$ is

$$\sim [\sim (p \wedge q) \vee (p \vee \sim q)] \equiv \sim [\sim (p \wedge q)] \wedge \sim (p \vee \sim q) \dots\dots\dots(\text{Negation of disjunction})$$

$$\equiv \sim [\sim (p \wedge q)] \wedge [\sim p \wedge \sim (\sim q)] \dots\dots\dots(\text{Negation of disjunction})$$

$$\equiv (p \wedge q) \wedge (\sim p \wedge q) \dots\dots\dots(\text{Negation of negation})$$

Exercise 1.4 | Q 1.7 | Page 21

Using the rule of negation write the negation of the following with justification.

$$(p \vee \sim q) \rightarrow (p \wedge \sim q)$$

Solution: The negation of $(p \vee \sim q) \rightarrow (p \wedge \sim q)$ is

$$\sim [(p \vee \sim q) \rightarrow (p \wedge \sim q)] \equiv (p \vee \sim q) \wedge \sim (p \wedge \sim q) \dots\dots\dots(\text{Negation of implication})$$

$$\equiv (p \vee \sim q) \wedge [\sim p \vee \sim (\sim q)] \dots\dots\dots(\text{Negation of conjunction})$$

$$\equiv (p \vee \sim q) \wedge (\sim p \vee q) \dots\dots\dots(\text{Negation of negation})$$

Exercise 1.4 | Q 1.8 | Page 21

Using the rule of negation write the negation of the following with justification.

$$(\sim p \vee \sim q) \vee (p \wedge \sim q)$$

Solution: The negation of $(\sim p \vee \sim q) \vee (p \wedge \sim q)$ is

$\sim [(\sim p \vee \sim q) \vee (p \wedge \sim q)] \equiv \sim (\sim p \vee \sim q) \wedge \sim (p \wedge \sim q)$ (Negation of disjunction)

$\equiv [\sim (\sim p) \wedge \sim (\sim q)] \wedge [\sim p \vee \sim (\sim q)]$...(Negation of disjunction and conjunction)

$\equiv (p \wedge q) \wedge (\sim p \vee q)$ (Negation of negation)

Exercise 1.4 | Q 2.1 | Page 21

Rewrite the following statement without using if then.

If a man is a judge then he is honest.

Solution: Since $p \rightarrow q \equiv \sim p \vee q$, the given statement can be written as:

A man is not a judge or he is honest.

Exercise 1.4 | Q 2.2 | Page 21

Rewrite the following statement without using if then.

If 2 is a rational number then $\sqrt{2}$ is irrational number.

Solution:

Since $p \rightarrow q \equiv \sim p \vee q$, the given statement can be written as:

2 is not a rational number or $\sqrt{2}$ is irrational number.

Exercise 1.4 | Q 2.3 | Page 21

Rewrite the following statement without using if then.

If $f(2) = 0$ then $f(x)$ is divisible by $(x - 2)$.

Solution: Since $p \rightarrow q \equiv \sim p \vee q$, the given statement can be written as:

$f(2) \neq 0$ or $f(x)$ is divisible by $(x - 2)$.

Exercise 1.4 | Q 3.1 | Page 21

Without using the truth table show that $P \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

Solution: L.H.S = $p \leftrightarrow q$

$\equiv (p \rightarrow q) \wedge (q \rightarrow p)$ (Biconditional Law)

$\equiv (\sim p \vee q) \wedge (\sim q \vee p)$ (Conditional Law)

$\equiv [\sim p \wedge (\sim q \vee p)] \vee [q \wedge (\sim q \vee p)]$ (Distributive Law)

$\equiv [(\sim p \wedge \sim q)] \vee (\sim p \wedge p) \vee [(q \wedge \sim q) \vee (q \wedge p)]$ (Distributive Law)

$$\equiv [(\sim p \wedge \sim q) \vee F] \vee [F \vee (q \wedge p)] \dots\dots\dots(\text{Complement Law})$$

$$\equiv (\sim p \wedge \sim q) \vee (q \wedge p) \dots\dots\dots(\text{Identity Law})$$

$$\equiv (\sim p \wedge \sim q) \vee (p \wedge q) \dots\dots\dots(\text{Commutative Law})$$

$$\equiv (p \wedge q) \vee (\sim p \wedge \sim q) \dots\dots\dots(\text{Commutative Law})$$

$$\equiv \text{R.H.S.}$$

Exercise 1.4 | Q 3.2 | Page 21

Without using truth table prove that:

$$(p \vee q) \wedge (p \vee \sim q) \equiv p$$

$$\textbf{Solution:} \text{ L.H.S.} = (p \vee q) \wedge (p \vee \sim q)$$

$$\equiv p \vee (q \wedge \sim q) \dots\dots\dots(\text{Distributive Law})$$

$$\equiv p \vee F \dots\dots\dots(\text{Complement Law})$$

$$\equiv p \dots\dots\dots(\text{Identity Law})$$

$$= \text{R.H.S.}$$

Exercise 1.4 | Q 3.3 | Page 21

Without using truth table prove that:

$$(p \wedge q) \vee (\sim p \wedge q) \vee (p \wedge \sim q) \equiv p \vee q$$

$$\textbf{Solution:} \text{ L.H.S.} = (p \wedge q) \vee (\sim p \wedge q) \vee (p \wedge \sim q)$$

$$\equiv [(p \vee \sim p) \wedge q] \vee (p \wedge \sim q) \dots\dots\dots(\text{Distributive Law})$$

$$\equiv (T \wedge q) \vee (p \wedge \sim q) \dots\dots\dots(\text{Complement Law})$$

$$\equiv q \vee (p \wedge \sim q) \dots\dots\dots(\text{Identity Law})$$

$$\equiv (q \vee p) \wedge (q \vee \sim q) \dots\dots\dots(\text{Distributive Law})$$

$$\equiv (q \vee p) \wedge T \dots\dots\dots(\text{Complement Law})$$

$$\equiv q \vee p \dots\dots\dots(\text{Identity Law})$$

$$\equiv p \vee q \dots\dots\dots(\text{Commutative Law})$$

$$= \text{R.H.S.}$$

Exercise 1.4 | Q 3.4 | Page 21

Without using truth table prove that:

$$\sim [(p \vee \sim q) \rightarrow (p \wedge \sim q)] \equiv (p \vee \sim q) \wedge (\sim p \vee q)$$

Solution: L.H.S. = $\sim [(p \vee \sim q) \rightarrow (p \wedge \sim q)]$

$$\equiv (p \vee \sim q) \rightarrow (p \wedge \sim q) \dots\dots(\text{Negation of implication})$$

$$\equiv (p \vee \sim q) \wedge [\sim p \vee \sim (\sim q)] \dots\dots(\text{Negation of conjunction})$$

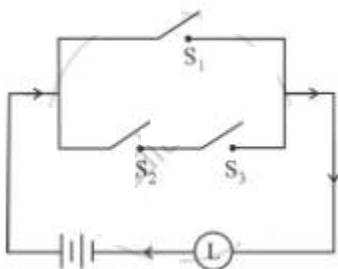
$$\equiv (p \vee \sim q) \wedge (\sim p \vee q) \dots\dots(\text{Negation of negation})$$

= R.H.S.

EXERCISE 1.5 [PAGES 29 - 30]

Exercise 1.5 | Q 1.1 | Page 29

Express the following circuit in the symbolic form of logic and write the input-output table.



Solution: Let p: the switch S_1 is closed

q: the switch S_2 is closed

r: the switch S_3 is closed

$\sim p$: the switch S_1' is closed or the switch S_1 is open

$\sim q$: the switch S_2' is closed or the switch S_2 is open

$\sim r$: the switch S_3' is closed or the switch S_3 is open

I: the lamp L is on

The symbolic form of the given circuit is:

$$p \vee (q \wedge r) \equiv I$$

I is generally dropped and it can be expressed as:

$$p \vee (q \wedge r)$$

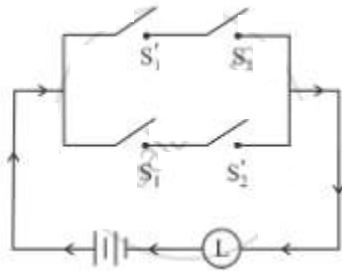
Input-Output Table

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$
1	1		1	1
1	1		0	1

1	0		0	1
1	0		0	1
0	1		1	1
0	1		0	0
0	0		0	0
0	0		0	0

Exercise 1.5 | Q 1.2 | Page 29

Express the following circuit in the symbolic form of logic and write the input-output table.



Solution: Let p: the switch S_1 is closed

q: the switch S_2 is closed

r: the switch S_3 is closed

$\sim p$: the switch S_1' is closed or the switch S_1 is open

$\sim q$: the switch S_2' is closed or the switch S_2 is open

$\sim r$: the switch S_3' is closed or the switch S_3 is open

l: the lamp L is on

The symbolic form of the given circuit is:

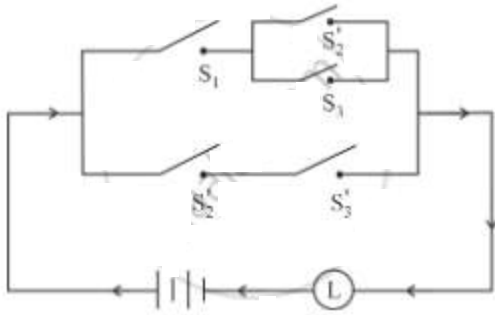
$$(\sim p \wedge q) \vee (p \wedge \sim q)$$

Input-Output Table

p	q	$\sim p$	$\sim q$	$\sim p \wedge q$	$p \wedge \sim q$	$(\sim p \wedge q) \vee (p \wedge \sim q)$
1	1	0	0	0	0	0
1	0	0	1	0	1	1
0	1	1	0	1	0	1
0	0	1	1	0	0	0

Exercise 1.5 | Q 1.3 | Page 29

Express the following circuit in the symbolic form of logic and write the input-output table.



Solution: Let p: the switch S_1 is closed

q: the switch S_2 is closed

r: the switch S_3 is closed

$\sim p$: the switch S_1' is closed or the switch S_1 is open

$\sim q$: the switch S_2' is closed or the switch S_2 is open

$\sim r$: the switch S_3' is closed or the switch S_3 is open

l: the lamp L is on

The symbolic form of the given circuit is:

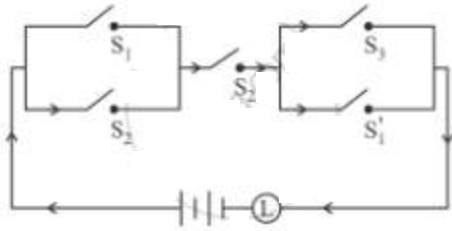
$$[p \wedge (\sim q \vee r)] \vee (\sim q \wedge \sim r)$$

Input-Output Table

p	q	r	$\sim q$	$\sim r$	$\sim q \vee r$	$p \wedge (\sim q \vee r)$	$\sim q \wedge \sim r$	$[p \wedge (\sim q \vee r)] \vee (\sim q \wedge \sim r)$
1	1				1	1	0	1
1	1				0	0	0	0
1	0				1	1	0	1
1	0				1	1	1	1
0	1				1	0	0	0
0	1				0	0	0	0
0	0				1	0	0	0
0	0				1	0	1	1

Exercise 1.5 | Q 1.4 | Page 29

Express the following circuit in the symbolic form of logic and write the input-output table.



Solution: Let p: the switch S_1 is closed

q: the switch S_2 is closed

r: the switch S_3 is closed

$\sim p$: the switch S_1' is closed or the switch S_1 is open

$\sim q$: the switch S_2' is closed or the switch S_2 is open

$\sim r$: the switch S_3' is closed or the switch S_3 is open

l: the lamp L is on

The symbolic form of the given circuit is:

$$(p \vee q) \wedge q \wedge (r \vee \sim p)$$

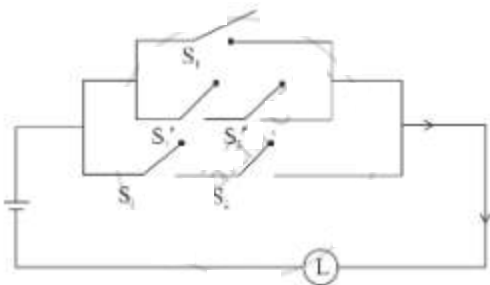
Input-Output Table

p	q	r	$\sim p$	$p \vee q$	$r \vee \sim p$	$(p \vee q) \wedge q \wedge (r \vee \sim p)$
1	1		0	1	1	1
1	1		0	1	0	0
1	0		0	1	1	0
1	0		0	1	0	0
0	1		1	1	1	1
0	1		1	1	1	1
0	0		1	0	1	0
0	0		1	0	1	0

[Note: Answer in the textbook is incorrect.]

Exercise 1.5 | Q 1.5 | Page 29

Express the following circuit in the symbolic form of logic and writ the input-output table.



Solution: Let p: the switch S_1 is closed

q: the switch S_2 is closed

r: the switch S_3 is closed

$\sim p$: the switch S_1' is closed or the switch S_1 is open

$\sim q$: the switch S_2' is closed or the switch S_2 is open

$\sim r$: the switch S_3' is closed or the switch S_3 is open

l: the lamp L is on

The symbolic form of the given circuit is:

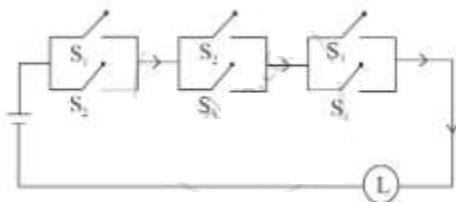
$$[p \vee (\sim p \wedge \sim q)] \vee (p \wedge q)$$

Input-Output Table

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$p \vee (\sim p \wedge \sim q)$	$p \wedge q$	$[p \vee (\sim p \wedge \sim q)] \vee (p \wedge q)$
1	1	0	0	0	1	1	1
1	0	0	1	0	1	0	1
0	1	1	0	0	0	0	0
0	0	1	1	1	1	0	1

Exercise 1.5 | Q 1.6 | Page 29

Express the following circuit in the symbolic form of logic and write the input-output table.



Solution: Let p: the switch S_1 is closed

q: the switch S_2 is closed

r : the switch S_3 is closed

$\sim p$: the switch S_1' is closed or the switch S_1 is open

$\sim q$: the switch S_2' is closed or the switch S_2 is open

$\sim r$: the switch S_3' is closed or the switch S_3 is open

I : the lamp L is on

The symbolic form of the given circuit is:

$$(p \vee q) \wedge (q \vee r) \wedge (r \vee p)$$

Input-Output Table

p	q	r	$p \vee q$	$q \vee r$	$r \vee p$	$(p \vee q) \wedge (q \vee r) \wedge (r \vee p)$
1	1	1	1	1	1	1
1	1	0	1	1	1	1
1	0	1	1	1	1	1
1	0	0	1	0	1	0
0	1	1	1	1	1	1
0	1	0	1	1	0	0
0	0	1	0	1	1	0
0	0	0	0	0	0	0

Exercise 1.5 | Q 2.1 | Page 30

Construct the switching circuit of the following:

$$(\sim p \wedge q) \vee (p \wedge \sim r)$$

Solution: Let p : the switch S_1 is closed

q : the switch S_2 is closed

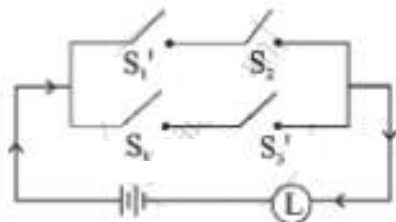
r : the switch S_3 is closed

$\sim p$: the switch S_1' is closed or the switch S_1 is open

$\sim q$: the switch S_2' is closed or the switch S_2 is open

$\sim r$: the switch S_3' is closed or the switch S_3 is open.

Then the switching circuit corresponding to the given statement pattern is:



Exercise 1.5 | Q 2.2 | Page 30

Construct the switching circuit of the following:

$$(p \wedge q) \vee [\sim p \wedge (\sim q \vee p \vee r)]$$

Solution: Let p: the switch S_1 is closed

q: the switch S_2 is closed

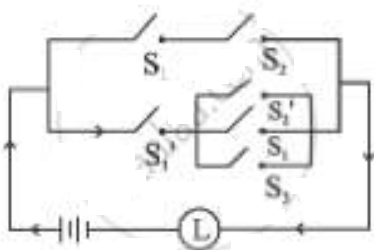
r: the switch S_3 is closed

$\sim p$: the switch S_1' is closed or the switch S_1 is open

$\sim q$: the switch S_2' is closed or the switch S_2 is open

$\sim r$: the switch S_3' is closed or the switch S_3 is open.

Then the switching circuit corresponding to the given statement pattern is:



Exercise 1.5 | Q 2.3 | Page 30

Construct the switching circuit of the following:

$$(p \wedge r) \vee (\sim q \wedge \sim r)] \wedge (\sim p \wedge \sim r)$$

Solution: Let p: the switch S_1 is closed

q: the switch S_2 is closed

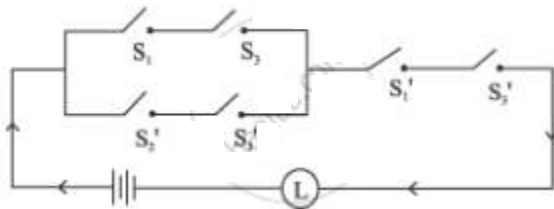
r: the switch S_3 is closed

$\sim p$: the switch S_1' is closed or the switch S_1 is open

$\sim q$: the switch S_2' is closed or the switch S_2 is open

$\sim r$: the switch S_3' is closed or the switch S_3 is open.

Then the switching circuit corresponding to the given statement pattern is:



Exercise 1.5 | Q 2.4 | Page 30

Construct the switching circuit of the following:

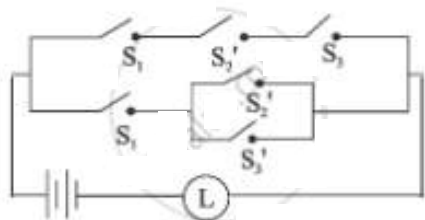
$$(p \wedge \sim q \wedge r) \vee [p \wedge (\sim q \vee \sim r)]$$

Solution: Let p: the switch S_1 is closed

q: the switch S_2 is closed

- r: the switch S_3 is closed
 $\sim p$: the switch S_1' is closed or the switch S_1 is open
 $\sim q$: the switch S_2' is closed or the switch S_2 is open
 $\sim r$: the switch S_3' is closed or the switch S_3 is open.

Then the switching circuit corresponding to the given statement pattern is:



Exercise 1.5 | Q 2.5 | Page 30

Construct the switching circuit of the following:

$$p \vee (\sim p) \vee (\sim q) \vee (p \wedge q)$$

Solution: Let p: the switch S_1 is closed

q: the switch S_2 is closed

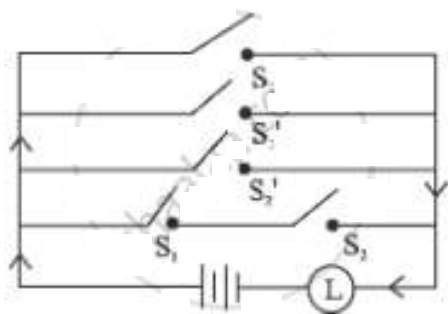
r: the switch S_3 is closed

$\sim p$: the switch S_1' is closed or the switch S_1 is open

$\sim q$: the switch S_2' is closed or the switch S_2 is open

$\sim r$: the switch S_3' is closed or the switch S_3 is open.

Then the switching circuit corresponding to the given statement pattern is:



Exercise 1.5 | Q 2.6 | Page 30

Construct the switching circuit of the following:

$$(p \wedge q) \vee (\sim p) \vee (p \wedge \sim q)$$

Solution: Let p: the switch S_1 is closed

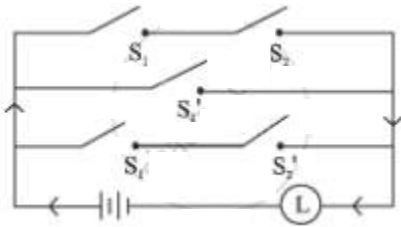
q: the switch S_2 is closed

r: the switch S_3 is closed

$\sim p$: the switch S_1' is closed or the switch S_1 is open

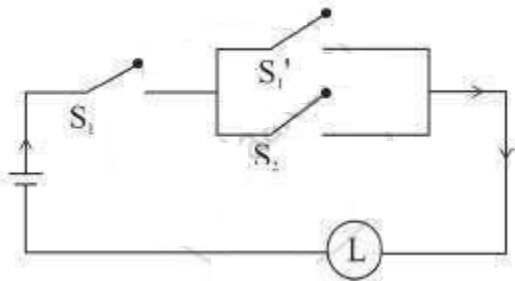
- ~ q: the switch S_2' is closed or the switch S_2 is open
- ~ r: the switch S_3' is closed or the switch S_3 is open.

Then the switching circuit corresponding to the given statement pattern is:



Exercise 1.5 | Q 3.1 | Page 30

Give an alternative equivalent simple circuit for the following circuit:



Solution: Let p: the switch S_1 is closed

q: the switch S_2 is closed

~ p: the switch S_1' is closed or the switch S_1 is open

Then the symbolic form of the given circuit is

$$p \wedge (\sim p \vee q).$$

Using the laws of logic, we have,

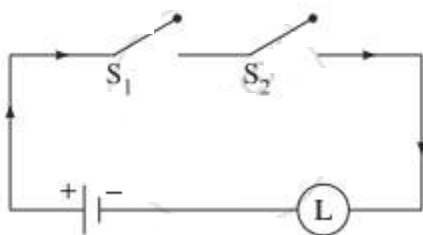
$$p \wedge (\sim p \vee q)$$

$$\equiv (p \wedge \sim p) \vee (p \wedge q) \dots\dots\dots(\text{By Distributive Law})$$

$$\equiv F \vee (p \wedge q) \dots\dots\dots(\text{By Complement Law})$$

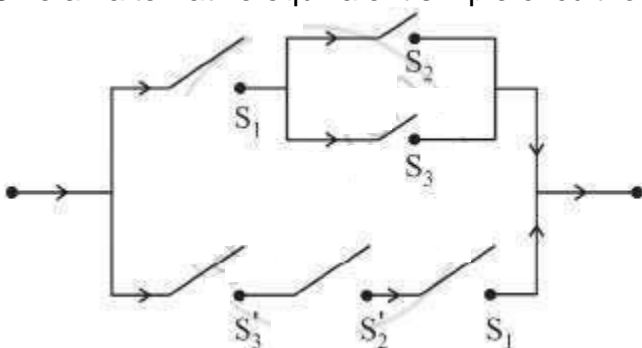
$$\equiv p \wedge q \dots\dots\dots(\text{By Identity Law})$$

Hence, the alternative equivalent simple circuit is:



Exercise 1.5 | Q 3.2 | Page 30

Give an alternative equivalent simple circuit for the following circuit:



Solution: Let p : the switch S_1 is closed

q : the switch S_2 is closed

r : the switch S_3 is closed

$\sim q$: the switch S_2' is closed or the switch S_2 is open

$\sim r$: the switch S_3' is closed or the switch S_3 is open.

Then the symbolic form of the given circuit is:

$$[p \wedge (q \vee r)] \vee (\sim r \wedge \sim q \wedge p)$$

Using the laws of logic, we have

$$[p \wedge (q \vee r)] \vee (\sim r \wedge \sim q \wedge p)$$

$$\equiv [p \wedge (q \vee r)] \vee [\sim (r \vee q) \wedge p] \dots\dots (\text{By De Morgan's Law})$$

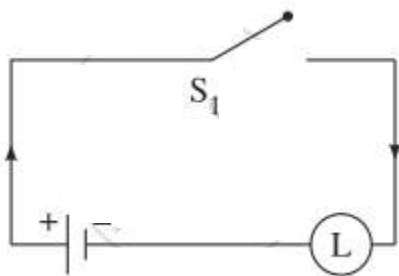
$$\equiv [p \wedge (q \vee r)] \vee [p \wedge \sim (q \vee r)] \dots\dots (\text{By Commutative Law})$$

$$\equiv p \wedge [(q \vee r) \vee \sim (q \vee r)] \dots\dots (\text{By Distributive Law})$$

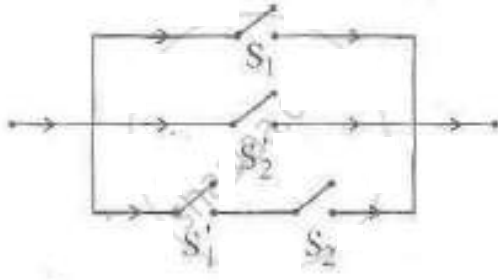
$$\equiv p \wedge T \dots\dots (\text{By Complement Law})$$

$$\equiv p \dots\dots (\text{By Identity Law})$$

Hence, the alternative equivalent simple circuit is


Exercise 1.5 | Q 4.1 | Page 30

find the symbolic form of the following switching circuit, construct its switching table and interpret it.



Solution: Let

p: The switch S_1 is closed,

q: The switch S_2 is closed.

Switching circuit is $(p \vee \sim q) \vee (\sim p \wedge q)$

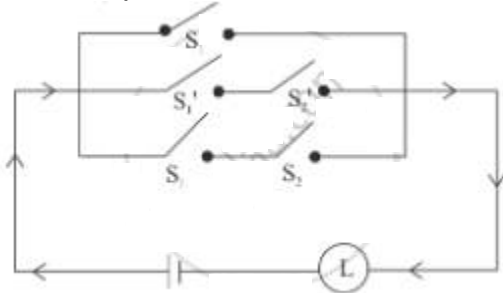
The switching table

p	q	$\sim p$	$\sim q$	$p \vee \sim q$	$\sim p \wedge q$	$(p \vee \sim q) \vee (\sim p \wedge q)$
1	1	0	0	1	0	1
1	0	0	1	1	0	1
0	1	1	0	0	1	1
0	0	1	1	1	0	1

From the last column of switching table we conclude that the current will always flow through the circuit.

Exercise 1.5 | Q 4.2 | Page 30

Write the symbolic form of the following switching circuit construct its switching table and interpret it.



Solution: Let p: the switch S_1 is closed

q: the switch S_2 is closed

$\sim p$: the switch S_1' is closed or the switch S_1 is open.

$\sim q$: the switch S_2' is closed or the switch S_2 is open.

Then the symbolic form of the given circuit is:

$$p \vee (\sim p \wedge \sim q) \vee (p \wedge q)$$

Switching Table

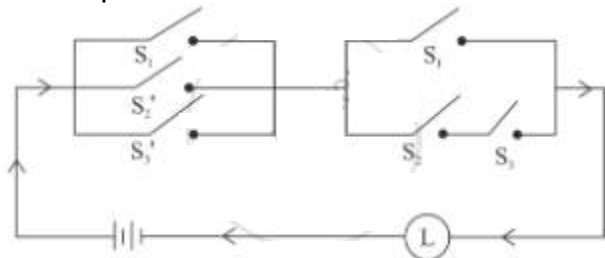
p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$p \wedge q$	$p \vee (\sim p \wedge \sim q) \vee (p \wedge q)$
1	1	0	0	0	1	1
1	0	0	1	0	0	1
0	1	1	0	0	0	0
0	0	1	1	1	0	1

Since the final column contains '0' when p is 0 and q is '1', otherwise it contains '1'.

Hence, the lamp will not glow when S_1 is OFF and S_2 is ON, otherwise, the lamp will glow.

Exercise 1.5 | Q 4.3 | Page 30

Write the symbolic form of the following switching circuit construct its switching table and interpret it.



Solution: Let p: the switch S_1 is closed

q: the switch S_2 is closed

r: the switch S_3 is closed

$\sim q$: the switch S_2' is closed or the switch S_2 is open

$\sim r$: the switch S_3' is closed or the switch S_3 is open

Then the symbolic form of the given circuit is:

$$[p \vee (\sim q) \vee (\sim r)] \wedge [p \vee (q \wedge r)]$$

Switching Table

p	q	r	$\sim q$	$\sim r$	$p \vee (\sim q) \vee (\sim r)$	$q \wedge r$	$p \vee (q \wedge r)$	Final column
					(I)		(II)	(I) \wedge (II)
1	1	1	0	0	1	1	1	1
1	1	0	0	1	1	0	1	1

1	0	1	1	0	1	0	1	1
1	0	0	1	1	1	0	1	1
0	1	1	0	0	0	1	1	0
0	1	0	0	1	1	0	0	0
0	0	1	1	0	1	0	0	0
0	0	0	1	1	1	0	0	0

From the switching table, the 'final column' and the column of p are identical. Hence, the lamp will glow which S_1 is 'ON'.

Exercise 1.5 | Q 5.1 | Page 30

Obtain the simple logical expression of the following. Draw the corresponding switching circuit.

$$p \vee (q \wedge \sim q)$$

Solution: Using the laws of logic, we have,

$$p \vee (q \wedge \sim q)$$

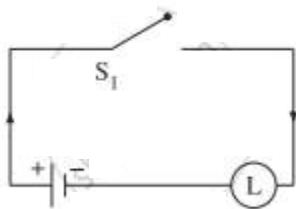
$$\equiv p \vee F \text{(By Complement Law)}$$

$$\equiv p \text{(By Identity Law)}$$

Hence, the simple logical expression of the given expression is p .

Let p : the switch S_1 is closed

Then the corresponding switching circuit is:



Exercise 1.5 | Q 5.2 | Page 30

Obtain the simple logical expression of the following. Draw the corresponding switching circuit.

$$(\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (p \wedge \sim q)$$

Solution: Using the laws of logic, we have,

$$(\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (p \wedge \sim q)$$

$$\equiv [\sim p \wedge (q \vee \sim q)] \vee (p \wedge \sim q) \text{(By Distributive Law)}$$

$$\equiv (\sim p \wedge T) \vee (p \wedge \sim q) \text{(By Complement Law)}$$

$$\equiv \sim p \vee (p \wedge \sim q) \text{(By Identity Law)}$$

$$\equiv (\sim p \vee p) \wedge (\sim p \wedge \sim q) \text{(By Distributive Law)}$$

$$\equiv T \wedge (\sim p \wedge \sim q) \text{(By Complement Law)}$$

$\equiv \sim p \vee \sim q$ (By Identity Law)

Hence, the simple logical expression of the given expression is $\sim p \vee \sim q$.

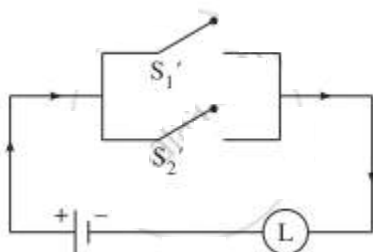
Let p: the switch S_1 is closed

q: the switch S_2 is closed

$\sim p$: the switch S_1' is closed or the switch S_1 is open

$\sim q$: the switch S_2' is closed or the switch S_2 is open.

Then the corresponding switching circuit is:



Exercise 1.5 | Q 5.3 | Page 30

Obtain the simple logical expression of the following. Draw the corresponding switching circuit.

$[p \vee (\sim q) \vee (\sim r)] \wedge [p \vee (q \wedge r)]$

Solution: Using the laws of logic, we have,

$[p \vee (\sim q) \vee (\sim r)] \wedge [p \vee (q \wedge r)]$

$\equiv [p \vee \{\sim (q \wedge r)\}] \wedge [p \vee (q \wedge r)]$ (By De Morgan's Law)

$\equiv p \vee [\sim (q \wedge r) \wedge (q \wedge r)]$ (By Distributive Law)

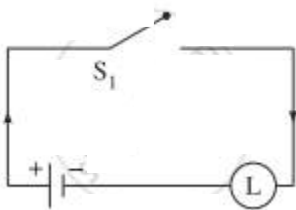
$\equiv p \vee F$ (By Complement Law)

$\equiv p$ (By Identity Law)

Hence, the simple logical expression of the given expression is p.

Let p: the switch S_1 is closed

Then the corresponding switching circuit is:



Exercise 1.5 | Q 5.4 | Page 30

Obtain the simple logical expression of the following. Draw the corresponding switching circuit.

$$(p \wedge q \wedge \sim p) \vee (\sim p \wedge q \wedge r) \vee (p \wedge q \wedge r)$$

Solution: Using the laws of logic, we have,

$$(p \wedge q \wedge \sim p) \vee (\sim p \wedge q \wedge r) \vee (p \wedge q \wedge r)$$

$$\equiv (p \wedge \sim p \wedge q) \vee (\sim p \wedge q \wedge r) \vee (p \wedge q \wedge r) \dots\dots\dots(\text{By Commutative Law})$$

$$\equiv (F \wedge q) \vee (\sim p \wedge q \wedge r) \vee (p \wedge q \wedge r) \dots\dots\dots(\text{By Complement Law})$$

$$\equiv F \vee (\sim p \wedge q \wedge r) \vee (p \wedge q \wedge r) \dots\dots\dots(\text{By Identity Law})$$

$$\equiv (\sim p \wedge q \wedge r) \vee (p \wedge q \wedge r) \dots\dots\dots(\text{By Identity Law})$$

$$\equiv (\sim p \vee p) \wedge (q \wedge r) \dots\dots\dots(\text{By Distributive Law})$$

$$\equiv T \wedge (q \wedge r) \dots\dots\dots(\text{By Complement Law})$$

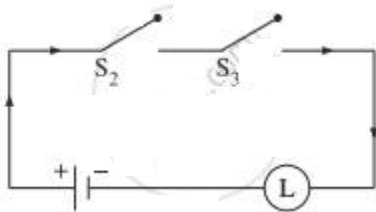
$$\equiv q \wedge r \dots\dots\dots(\text{By Identity Law})$$

Hence, the simple logical expression of the given expression is $q \wedge r$.

Let q : the switch S_2 is closed

r : the switch S_3 is closed.

Then the corresponding switching circuit is:



MISCELLANEOUS EXERCISE 1 [PAGES 32 - 35]

Miscellaneous Exercise 1 | Q 1.1 | Page 32

Select and write the correct answer from the given alternative of the following question:

If $p \wedge q$ is false and $p \vee q$ is true, then _____ is not true.

1. $p \vee q$
2. $p \leftrightarrow q$
3. $\sim p \vee \sim q$
4. $q \vee \sim p$

Solution: If $p \wedge q$ is false and $p \vee q$ is true, then $p \leftrightarrow q$ is not true.

Miscellaneous Exercise 1 | Q 1.2 | Page 32

Select and write the correct answer from the given alternative of the following question:

$(p \wedge q) \rightarrow r$ is logically equivalent to _____ .

1. $p \rightarrow (q \rightarrow r)$
2. $(p \wedge q) \rightarrow \sim r$
3. $(\sim p \vee \sim q) \rightarrow \sim r$
4. $(p \vee q) \rightarrow r$

Solution: $(p \wedge q) \rightarrow r$ is logically equivalent to $p \rightarrow (q \rightarrow r)$.

Miscellaneous Exercise 1 | Q 1.3 | Page 32

Select and write the correct answer from the given alternative of the following question:

Inverse of statement pattern $(p \vee q) \rightarrow (p \wedge q)$ is _____ .

1. $(p \wedge q) \rightarrow (p \vee q)$
2. $\sim (p \vee q) \rightarrow (p \wedge q)$
3. $(\sim p \wedge \sim q) \rightarrow (\sim p \vee \sim q)$
4. $(\sim p \vee \sim q) \rightarrow (\sim p \wedge \sim q)$

Solution: Inverse of statement pattern $(p \vee q) \rightarrow (p \wedge q)$ is $(\sim p \wedge \sim q) \rightarrow (\sim p \vee \sim q)$.

Miscellaneous Exercise 1 | Q 1.4 | Page 32

Select and write the correct answer from the given alternative of the following question:

If $p \wedge q$ is F, $p \rightarrow q$ is F then the truth values of p and q are _____.

1. T, T
2. T, F
3. F, T
4. F, F

Solution: If $p \wedge q$ is F, $p \rightarrow q$ is F then the truth values of p and q are T, F.

Miscellaneous Exercise 1 | Q 1.5 | Page 32

Select and write the correct answer from the given alternative of the following question:

The negation of inverse of $\sim p \rightarrow q$ is _____.

1. $q \wedge p$
2. $\sim p \wedge \sim q$
3. $p \wedge q$

4. $\sim q \rightarrow \sim p$

Solution: The negation of inverse of $\sim p \rightarrow q$ is $q \wedge p$.

Miscellaneous Exercise 1 | Q 1.6 | Page 32

Select and write the correct answer from the given alternative of the following question:

The negation of $p \wedge (q \rightarrow r)$ is _____.

1. $\sim p \wedge (\sim q \rightarrow \sim r)$
2. $p \vee (\sim q \vee r)$
3. $\sim p \wedge (\sim q \rightarrow \sim r)$
4. $\sim p \vee (\sim q \wedge \sim r)$

Solution: $\forall x \in A, x + 6 \geq 9$

Miscellaneous Exercise 1 | Q 1.7 | Page 32

Select and write the correct answer from the given alternative of the following question:

If $A = \{1, 2, 3, 4, 5\}$ then which of the following is not true?

1. $\exists x \in A$ such that $x + 3 = 8$
2. $\exists x \in A$ such that $x + 2 < 9$
3. $\forall x \in A, x + 6 \geq 9$
4. $\exists x \in A$ such that $x + 6 < 10$

Solution: $\forall x \in A, x + 6 \geq 9$

Miscellaneous Exercise 1 | Q 2.1 | Page 33

Which of the following sentence is the statement in logic? Justify. Write down the truth value of the statement:

$4! = 24$.

Solution: It is a statement which is true, hence its truth value is 'T'.

Miscellaneous Exercise 1 | Q 2.2 | Page 33

Which of the following sentence is the statement in logic? Justify. Write down the truth value of the statement:

π is an irrational number.

Solution: It is a statement which is true, hence its truth value is 'T'.

Miscellaneous Exercise 1 | Q 2.3 | Page 33

Which of the following sentence is the statement in logic? Justify. Write down the truth value of the statement:

India is a country and Himalayas is a river.

Solution: It is a statement which is false, hence its truth value is 'F'.[$T \wedge F \equiv F$]

Miscellaneous Exercise 1 | Q 2.4 | Page 33

Which of the following sentence is the statement in logic? Justify. Write down the truth value of the statement:

Please get me a glass of water.

Solution: It is an imperative sentence, hence it is not a statement.

Miscellaneous Exercise 1 | Q 2.5 | Page 33

Which of the following sentence is the statement in logic? Justify. Write down the truth value of the statement:

$\cos^2\theta - \sin^2\theta = \cos 2\theta$ for all $\theta \in \mathbb{R}$.

Solution: It is a statement which is true, hence its truth value is 'T'.

Miscellaneous Exercise 1 | Q 2.6 | Page 33

Which of the following sentence is the statement in logic? Justify. Write down the truth value of the statement:

If x is a whole number then $x + 6 = 0$.

Solution: It is a statement which is false, hence its truth value is 'F'.

[Note: Answer in the textbook is incorrect.]

Miscellaneous Exercise 1 | Q 3.1 | Page 33

Write the truth value of the following statement:

$\sqrt{5}$ is an irrational but $3\sqrt{5}$ is a complex number.

Solution: Let p : 5 is an irrational.

q : $3\sqrt{5}$ is a complex number.

Then the symbolic form of the given statement is $p \wedge q$.

The truth values of p and q are T and F respectively.

\therefore The truth value of $p \wedge q$ is F.[$T \wedge F \equiv F$]

[Note: Answer in the textbook is incorrect.]

Miscellaneous Exercise 1 | Q 3.2 | Page 33

Write the truth value of the following statement:

$\forall n \in \mathbb{N}$, $n^2 + n$ is even number while $n^2 - n$ is an odd number.

Solution: Let p : $\forall n \in \mathbb{N}$, $n^2 + n$ is an even number.

q : $\forall n \in \mathbb{N}$, $n^2 - n$ is an odd number.

Then the symbolic form of the given statement is $p \wedge q$.

The truth values of p and q are T and F respectively. \therefore The truth value of $p \wedge q$ is F.

.....[$T \wedge F \equiv F$].

Miscellaneous Exercise 1 | Q 3.3 | Page 33

Write the truth value of the following statement:

$\exists n \in \mathbb{N}$ such that $n + 5 > 10$.

Solution: $\exists n \in \mathbb{N}$, such that $n + 5 > 10$ is a true statement, hence its truth value is T.

(All $n \geq 6$, where $n \in \mathbb{N}$, satisfy $n + 5 > 10$).

Miscellaneous Exercise 1 | Q 3.4 | Page 33

Write the truth value of the following statement:

The square of any even number is odd or the cube of any odd number is odd.

Solution: Let p : The square of any even number is odd.

q : The cube of any odd number is odd.

Then the symbolic form of the given statement is $p \vee q$.

The truth values of p and q are F and T respectively.

\therefore The truth value of $p \vee q$ is T.[$F \vee T \equiv T$]

Miscellaneous Exercise 1 | Q 3.5 | Page 33

Write the truth value of the following statement:

In $\triangle ABC$ if all sides are equal then its all angles are equal.

Solution: Let p : ABC is a triangle and all its sides are equal.

q : It's all angles are equal.

Then the symbolic form of the given statement is $p \rightarrow q$.

If the truth value of p is T, then the truth value of q is T.

\therefore The truth value of $p \rightarrow q$ is T[$T \rightarrow T \equiv T$].

Miscellaneous Exercise 1 | Q 3.6 | Page 33

Write the truth value of the following statement:

$$\forall n \in \mathbb{N}, n + 6 > 8.$$

Solution: $\forall n \in \mathbb{N}, n + 6 > 8$ is a false statement, hence its truth value is F.

($n = 1 \in \mathbb{N}, n = 2 \in \mathbb{N}$ do not satisfy $n + 6 > 8$).

Miscellaneous Exercise 1 | Q 4.1 | Page 33

If $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, determine the truth value of the following statement:

$$\exists x \in A \text{ such that } x + 8 = 15$$

Solution: Clearly $x = 7 \in A$ satisfies $x + 8 = 15$. So the given statement is true, hence its truth value is T.

Miscellaneous Exercise 1 | Q 4.2 | Page 33

If $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, determine the truth value of the following statement:

$$\forall x \in A, x + 5 < 12.$$

Solution: There is no $x \in A$ which satisfies $x + 5 < 12$. So the given statement is false, hence its truth value is F.

Miscellaneous Exercise 1 | Q 4.3 | Page 33

If $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, determine the truth value of the following statement:

$$\exists x \in A, \text{ such that } x + 7 \geq 11.$$

Solution: Clearly $x = 1 \in A, x = 2 \in A$ and $x = 3 \in A$ satisfies $x + 7 \geq 11$. So the given statement is true, hence its truth value is T.

Miscellaneous Exercise 1 | Q 4.4 | Page 33

If $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, determine the truth value of the following statement:

$$\forall x \in A, 3x \leq 25.$$

Solution: $x = 9 \in A$ does not satisfy $3x \leq 25$ So the given statement is false, hence its truth value is F.

Miscellaneous Exercise 1 | Q 5.1 | Page 33

Write the negation of the following:

$$\forall n \in A, n + 7 > 6.$$

Solution: The negation of the given statement is:

$$\exists n \in A, \text{ such that } n + 7 \leq 6.$$

OR

$$\exists n \in A, \text{ such that } n + 7 \nless 6.$$

Miscellaneous Exercise 1 | Q 5.2 | Page 33

Write the negation of the following:

$$\exists x \in A, \text{ such that } x + 9 \leq 15.$$

Solution: The negation of the given statement is:

$$\forall x \in A, x + 9 > 15.$$

Miscellaneous Exercise 1 | Q 5.3 | Page 33

Write the negation of the following:

Some triangles are equilateral triangle.

Solution: The negation of the given statement is:

All triangles are not equilateral triangles.

Miscellaneous Exercise 1 | Q 6.1 | Page 33

Construct the truth table of the following:

$$p \rightarrow (q \rightarrow p)$$

Solution:

p	q	$q \rightarrow p$	$p \rightarrow (q \rightarrow p)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

Miscellaneous Exercise 1 | Q 6.2 | Page 33

Construct the truth table of the following:

$$(\sim p \vee \sim q) \leftrightarrow [\sim(p \wedge q)]$$

Solution:

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$	$p \wedge q$	$\sim (p \wedge q)$	$(\sim p \vee \sim q) \leftrightarrow [\sim (p \wedge q)]$
T	T	F	F	F	T	F	T
T	F	F	T	T	F	T	T
F	T	T	F	T	F	T	T
F	F	T	T	T	F	T	T

Miscellaneous Exercise 1 | Q 6.3 | Page 33

Construct the truth table of the following:

$$\sim (\sim p \wedge \sim q) \vee q$$

Solution:

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$\sim (\sim p \wedge \sim q)$	$\sim (\sim p \wedge \sim q) \vee q$
T	T	F	F	F	T	T
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	T	F	F

[Note: Answer in the textbook is incorrect.]

Miscellaneous Exercise 1 | Q 6.4 | Page 33

Construct the truth table of the following:

$$[(p \wedge q) \vee r] \wedge [\sim r \vee (p \wedge q)]$$

Solution:

p	q	r	$p \wedge q$	$(p \wedge q) \vee r$	$\sim r$	$\sim r \vee (p \wedge q)$	$[(p \wedge q) \vee r] \wedge [\sim r \vee (p \wedge q)]$
T	T	T	T	T	F	T	T
T	T	F	T	T	T	T	T
T	F	T	F	T	F	F	F
T	F	F	F	F	T	T	F
F	T	T	F	T	F	F	F
F	T	F	F	F	T	T	F
F	F	T	F	T	F	F	F
F	F	F	F	F	T	T	F

Miscellaneous Exercise 1 | Q 6.5 | Page 33

Construct the truth table of the following:

$$[(\sim p \vee q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$$

Solution:

p	q	r	$\sim p$	$\sim p \vee q$	$q \rightarrow r$	$(\sim p \vee q) \wedge (q \rightarrow r)$	$p \rightarrow r$	$[(\sim p \vee q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
T	T	T	F	T	T	T	T	T
T	T	F	F	T	F	F	F	T
T	F	T	F	F	T	F	T	T
T	F	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	F	F	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	T	T	T	T

Miscellaneous Exercise 1 | Q 7.1 | Page 33

Examine whether the following statement pattern is a tautology or a contradiction or a contingency.

$$[(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$$

Solution:

p	q	$\sim p$	$\sim q$	$p \rightarrow q$	$(p \rightarrow q) \wedge \sim q$	$[(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	F	T
F	F	T	T	T	T	T

All the entries in the last column of the above truth table are T.

$\therefore [(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$ is a tautology.

Miscellaneous Exercise 1 | Q 7.2 | Page 33

Determine whether the following statement pattern is a tautology, contradiction, or contingency:

$$[(p \vee q) \wedge \sim p] \wedge \sim q$$

Solution:

p	q	$\sim p$	$\sim q$	$p \vee q$	$(p \vee q) \wedge \sim p$	$[(p \vee q) \wedge \sim p] \wedge \sim q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	T	F
F	T	T	T	F	F	F

All the entries in the last column of the above truth table are F.

$\therefore [(p \vee q) \wedge \sim p] \wedge \sim q$ is a contradiction.

Miscellaneous Exercise 1 | Q 7.3 | Page 33

Determine whether the following statement pattern is a tautology, contradiction, or contingency:

$$(p \rightarrow q) \wedge (p \wedge \sim q)$$

Solution:

p	q	$\sim q$	$p \rightarrow q$	$p \wedge \sim q$	$(p \rightarrow q) \wedge (p \wedge \sim q)$
T	T	F	T	F	F
T	F	T	F	T	F
F	T	F	T	F	F
F	F	T	T	F	F

All the entries in the last column of the above truth table are F.

$\therefore (p \rightarrow q) \wedge (p \wedge \sim q)$ is a contradiction.

Miscellaneous Exercise 1 | Q 7.4 | Page 33

Determine whether the following statement pattern is a tautology, contradiction or contingency:

$$[p \rightarrow (q \rightarrow r)] \leftrightarrow [(p \wedge q) \rightarrow r]$$

Solution:

p	q	r	$q \rightarrow r$	$p \rightarrow (q \rightarrow r)$	$p \wedge q$	$(p \wedge q) \rightarrow r$	$[p \rightarrow (q \rightarrow r)] \leftrightarrow [(p \wedge q) \rightarrow r]$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	T
T	F	T	T	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	T	T	F	T	T
F	T	F	F	T	F	T	T
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

All the entries in the last column of the above truth table are T.

$\therefore [p \rightarrow (q \rightarrow r)] \leftrightarrow [(p \wedge q) \rightarrow r]$ is a tautology.

Miscellaneous Exercise 1 | Q 7.5 | Page 33

Determine whether the following statement pattern is a tautology, contradiction or contingency:

$$[(p \wedge (p \rightarrow q)) \rightarrow q]$$

Solution:

p	q	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	$[p \wedge (p \rightarrow q)] \rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

All the entries in the last column of the above truth table are T.

$\therefore [(p \wedge (p \rightarrow q)) \rightarrow q]$ is a tautology.

Miscellaneous Exercise 1 | Q 7.6 | Page 33

Determine whether the following statement pattern is a tautology, contradiction or contingency:

$$(p \wedge q) \vee (\sim p \wedge q) \vee (p \vee \sim q) \vee (\sim p \wedge \sim q)$$

Solution:

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim p \wedge q$	$p \vee \sim q$	$\sim p \wedge \sim q$	(I) \vee (II) \vee (III) \vee (IV)
				(I)	(II)	(III)	(IV)	
T	T	F	F	T	F	T	F	T
T	F	F	T	F	F	T	F	T
F	T	T	F	F	T	F	F	T
F	F	T	T	F	F	T	T	T

All the entries in the last column of the above truth table are T.

$\therefore (p \wedge q) \vee (\sim p \wedge q) \vee (p \vee \sim q) \vee (\sim p \wedge \sim q)$ is a tautology.

Miscellaneous Exercise 1 | Q 7.7 | Page 33

Determine whether the following statement pattern is a tautology, contradiction or contingency:

$$[(p \vee \sim q) \vee (\sim p \wedge q)] \wedge r$$

Solution:

p	q	r	$\sim p$	$\sim q$	$p \vee \sim q$	$\sim p \wedge q$	$(p \vee \sim q) \vee (\sim p \wedge q)$	(I) \wedge r
							(I)	
T	T	T	F	F	T	F	T	T
T	T	F	F	F	T	F	T	F
T	F	T	F	T	T	F	T	T
T	F	F	F	T	T	F	T	F
F	T	T	T	F	F	T	T	T
F	T	F	T	F	F	T	T	F
F	F	T	T	T	T	F	T	T
F	F	F	T	T	T	F	T	F

The entries in the last column are neither all T nor all F.

$\therefore [(p \vee \sim q) \vee (\sim p \wedge q)] \wedge r$ is a contingency.

Miscellaneous Exercise 1 | Q 7.8 | Page 33

Determine whether the following statement pattern is a tautology, contradiction or contingency:

$$(p \rightarrow q) \vee (q \rightarrow p)$$

Solution:

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \vee (q \rightarrow p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

All the entries in the last column of the above truth table are T.

$\therefore (p \rightarrow q) \vee (q \rightarrow p)$ is a tautology.

Miscellaneous Exercise 1 | Q 8.1 | Page 34

Determine the truth values of p and q in the following case:

$(p \vee q)$ is T and $(p \wedge q)$ is T

Solution:

p	q	$p \vee q$	$p \wedge q$
T	T	T	T
T	F	T	F
F	T	T	F
F	F	F	F

Since $p \vee q$ and $p \wedge q$ both are T, from the table, the truth values of both p and q are T.

Miscellaneous Exercise 1 | Q 8.2 | Page 34

Determine the truth values of p and q in the following case:

$(p \vee q)$ is T and $(p \vee q) \rightarrow q$ is F

Solution:

p	q	$p \vee q$	$(p \vee q) \rightarrow q$
T	T	T	T

T	F	T	F
F	T	T	T
F	F	F	T

Since the truth values of $(p \vee q)$ is T and $(p \vee q) \rightarrow q$ is F, from the table, the truth values of p and q are T and F respectively.

Miscellaneous Exercise 1 | Q 8.3 | Page 34

Determine the truth values of p and q in the following case:

$(p \wedge q)$ is F and $(p \wedge q) \rightarrow q$ is T

Solution:

p	q	$p \wedge q$	$(p \wedge q) \rightarrow q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Since the truth values of $(p \wedge q)$ is F and $(p \wedge q) \rightarrow q$ is T, from the table, the truth values of p and q are either T and F respectively or F and T respectively or both F.

Miscellaneous Exercise 1 | Q 9.1 | Page 34

Using the truth table, prove the following logical equivalence :

$$p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$$

Solution:

1	2	3	4	5	6	7	8
			A			B	
p	q	$p \leftrightarrow q$	$p \wedge q$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$A \vee B$
T	T	T	T	F	F	F	T
T	F	F	F	F	T	F	F
F	T	F	F	T	F	F	F
F	F	T	F	T	T	T	T

By column number 3 and 8

$$p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$$

Miscellaneous Exercise 1 | Q 9.2 | Page 34

Using truth table, prove the following logical equivalence :

$$(p \wedge q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$$

Solution:

1	2	3	4	5	6	7
p	q	r	$p \wedge q$	$(p \wedge q) \rightarrow r$	$q \rightarrow r$	$p \rightarrow (q \rightarrow r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	F	T	T	T
F	T	F	F	T	F	T
F	F	T	F	T	T	T
F	F	F	F	T	T	T

The entries in columns 5 and 7 are identical.

$$\therefore (p \wedge q) \rightarrow r \equiv p \rightarrow (q \rightarrow r).$$

Miscellaneous Exercise 1 | Q 10.1 | Page 34

Using rules in logic, prove the following:

$$p \leftrightarrow q \equiv \sim(p \wedge \sim q) \vee \sim(q \wedge \sim p)$$

Solution: By the rules of negation of biconditional,

$$\sim(p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$$

$$\therefore \sim[(p \wedge \sim q) \wedge (q \wedge \sim p)] \equiv p \leftrightarrow q$$

$$\therefore \sim(p \wedge \sim q) \wedge \sim(q \wedge \sim p) \equiv p \leftrightarrow q \dots\dots(\text{Negation of disjunction})$$

$$\therefore p \leftrightarrow q \equiv \sim(p \wedge \sim q) \wedge \sim(q \wedge \sim p).$$

Miscellaneous Exercise 1 | Q 10.2 | Page 34

Using rules in logic, prove the following:

$$\sim p \wedge q \equiv (p \vee q) \wedge \sim p$$

$$\text{Solution: } (p \vee q) \wedge \sim p$$

$$\equiv (p \wedge \sim p) \vee (q \wedge \sim p) \dots\dots(\text{Distributive Law})$$

$$\equiv F \vee (q \wedge \sim p) \dots\dots\dots(\text{Complement Law})$$

$$\equiv q \wedge \sim p \dots\dots\dots(\text{Identity Law})$$

$$\equiv \sim p \wedge q \dots\dots\dots(\text{Commutative Law})$$

$$\therefore \sim p \wedge q \equiv (p \vee q) \wedge \sim p$$

Miscellaneous Exercise 1 | Q 10.3 | Page 34

Using rules in logic, prove the following:

$$\sim (p \vee q) \vee (\sim p \wedge q) \equiv \sim p$$

Solution: $\sim (p \vee q) \vee (\sim p \wedge q)$

$$\equiv (\sim p \wedge \sim q) \vee (\sim p \wedge q) \dots\dots\dots(\text{Negation of disjunction})$$

$$\equiv \sim p \wedge (\sim q \vee q) \dots\dots\dots(\text{Distributive Law})$$

$$\equiv \sim p \wedge T \dots\dots\dots(\text{Complement Law})$$

$$\equiv \sim p \dots\dots\dots(\text{Identity Law})$$

$$\therefore \sim (p \vee q) \vee (\sim p \wedge q) \equiv \sim p$$

Miscellaneous Exercise 1 | Q 11.1 | Page 34

Using the rules in logic, write the negation of the following:

$$(p \vee q) \wedge (q \vee \sim r)$$

Solution: The negation of $(p \vee q) \wedge (q \vee \sim r)$ is

$$\sim [(p \vee q) \wedge (q \vee \sim r)]$$

$$\equiv \sim(p \vee q) \vee \sim(q \vee \sim r) \dots\dots(\text{Negation of conjunction})$$

$$\equiv (\sim p \wedge \sim q) \vee [\sim q \wedge \sim(\sim r)] \dots\dots\dots(\text{Negation of disjunction})$$

$$\equiv (\sim p \wedge \sim q) \vee (\sim q \wedge r) \dots\dots\dots(\text{Negation of negation})$$

$$\equiv (\sim q \wedge \sim p) \vee (\sim q \wedge r) \dots\dots\dots(\text{Commutative law})$$

$$\equiv (\sim q) \wedge (\sim p \vee r) \dots\dots\dots(\text{Distributive Law})$$

Miscellaneous Exercise 1 | Q 11.2 | Page 34

Using the rules in logic, write the negation of the following:

$$p \wedge (q \vee r)$$

Solution: The negation of $p \wedge (q \vee r)$ is

$$\sim [p \wedge (q \vee r)]$$

$$\equiv \sim p \vee \sim(q \vee r) \dots\dots\dots(\text{Negation of conjunction})$$

$$\equiv \sim p \vee (\sim q \wedge \sim r) \dots\dots\dots(\text{Negation of disjunction})$$

Miscellaneous Exercise 1 | Q 11.3 | Page 34

Using the rules in logic, write the negation of the following:

$$(p \rightarrow q) \wedge r$$

Solution: The negation of $(p \rightarrow q) \wedge r$ is

$$\sim [(p \rightarrow r) \wedge r]$$

$$\equiv \sim (p \rightarrow q) \vee (\sim r) \dots\dots\dots(\text{Negation of conjunction})$$

$$\equiv (p \wedge \sim q) \vee (\sim r) \dots\dots\dots(\text{Negation of implication})$$

[Note: Answer in the textbook is incorrect.]

Miscellaneous Exercise 1 | Q 11.4 | Page 34

Using the rules in logic, write the negation of the following:

$$(\sim p \wedge q) \vee (p \wedge \sim q)$$

Solution: The negation of $(\sim p \wedge q) \vee (p \wedge \sim q)$ is

$$\sim [(\sim p \wedge q) \vee (p \wedge \sim q)]$$

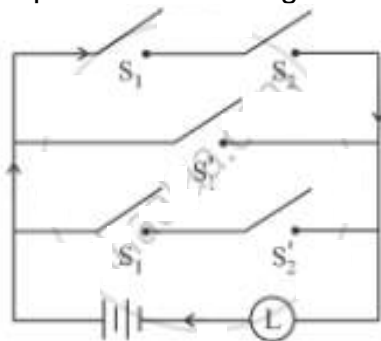
$$\equiv \sim (p \wedge q) \wedge \sim (p \wedge \sim q) \dots\dots(\text{Negation of disjunction})$$

$$\equiv [\sim (\sim p) \vee \sim q] \wedge [\sim p \vee \sim(\sim q)] \dots\dots\dots(\text{Negation of conjunction})$$

$$\equiv (p \vee \sim q) \wedge (\sim p \vee q) \dots\dots\dots(\text{Negation of negation})$$

Miscellaneous Exercise 1 | Q 12.1 | Page 34

Express the following circuit in the symbolic form. Prepare the switching table:



Solution:

Let p: the switch S_1 is closed

q: the switch S_2 is closed

$\sim p$: the switch S_1' is closed or the switch S_1 is open

$\sim q$: the switch S_2' is closed or the switch S_2 is open.

Then the symbolic form of the given circuit is:

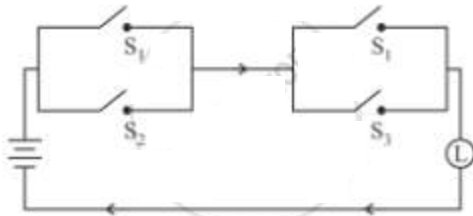
$$(p \wedge q) \vee (\sim p) \vee (p \wedge \sim q)$$

Switching Table

p	q	$\sim p$	$\sim q$	$p \wedge q$	$p \wedge \sim q$	$(p \wedge q) \vee (\sim p) \vee (p \wedge \sim q)$
1	1	0	0	1	0	1
1	0	0	1	0	1	1
0	1	1	0	0	0	1
0	0	1	1	0	0	1

Miscellaneous Exercise 1 | Q 12.2 | Page 34

Express the following circuit in the symbolic form. Prepare the switching table:



Solution: Let p: the switch S_1 is closed

q: the switch S_2 is closed

r: the switch S_3 is closed

Then the symbolic form of the given statement is:

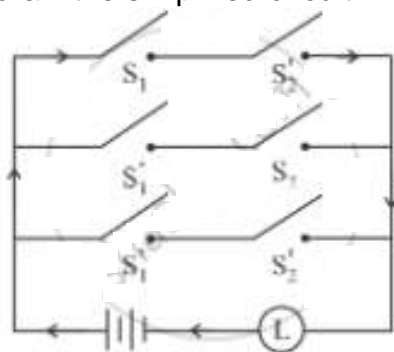
$$(p \vee q) \wedge (p \vee r)$$

Switching Table

p	q	r	$p \vee q$	$p \vee r$	$(p \vee q) \wedge (p \vee r)$
1	1	1	1	1	1
1	1	0	1	1	1
1	0	1	1	1	1
1	0	0	1	1	1
0	1	1	1	1	1
0	1	0	1	0	0
0	0	1	0	1	0
0	0	0	0	0	0

Miscellaneous Exercise 1 | Q 13.1 | Page 34

Simplify the following so that the new circuit has a minimum number of switches. Also, draw the simplified circuit.



Solution: Let p : the switch S_1 is closed

q : the switch S_2 is closed

$\sim p$: the switch S_1' is closed or the switch S_1 is open

$\sim q$: the switch S_2' is closed or the switch S_2 is open.

Then the given circuit in symbolic form is:

$$(p \wedge q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$$

Using the laws of logic, we have,

$$(p \wedge \sim q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$$

$$\equiv (p \wedge \sim q) \vee [(\sim p \wedge q) \vee (\sim p \wedge \sim q)] \dots\dots\dots (\text{By Associative Law})$$

$$\equiv (p \wedge \sim q) \vee [\sim p \wedge (q \vee \sim q)] \dots\dots\dots (\text{By Distributive Law})$$

$$\equiv (p \wedge \sim q) \vee (\sim p \wedge T) \dots\dots\dots (\text{By Complement Law})$$

$$\equiv (p \wedge \sim q) \vee \sim p \dots\dots\dots (\text{By Identity Law})$$

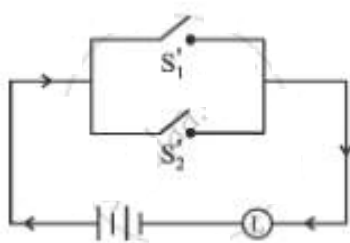
$$\equiv (p \vee \sim p) \wedge (\sim q \vee \sim p) \dots\dots\dots (\text{By Distributive Law})$$

$$\equiv T \wedge (\sim q \vee \sim p) \dots\dots\dots (\text{By Complement Law})$$

$$\equiv \sim q \vee \sim p \dots\dots\dots (\text{By Identity Law})$$

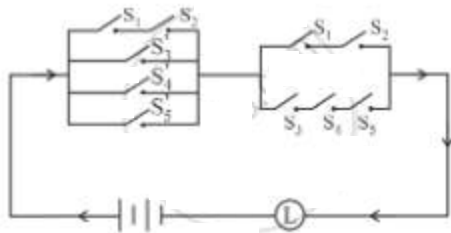
$$\equiv \sim p \vee \sim q \dots\dots\dots (\text{By Commutative Law})$$

Hence, the simplified circuit for the given circuit is:



Miscellaneous Exercise 1 | Q 13.2 | Page 34

Simplify the following so that the new circuit has a minimum number of switches. Also, draw the simplified circuit.



Solution: Let p: the switch S_1 is closed

q: the switch S_2 is closed

r: the switch S_3 is closed

s: the switch S_4 is closed

t: the switch S_5 is closed

$\sim p$: the switch S_1' is closed or the switch S_1 is open

$\sim q$: the switch S_2' is closed or the switch S_2 is open

$\sim r$: the switch S_3' is closed or the switch S_3 is open

$\sim s$: the switch S_4' is closed or the switch S_4 is open

$\sim t$: the switch S_5' is closed or the switch S_5 is open.

Then the given circuit in symbolic form is:

$$[(p \wedge q) \vee \sim r \vee \sim s \vee \sim t] \wedge [(p \wedge q) \vee (r \wedge s \wedge t)]$$

Using the laws of logic, we have,

$$[(p \wedge q) \vee \sim r \vee \sim s \vee \sim t] \wedge [(p \wedge q) \vee (r \wedge s \wedge t)]$$

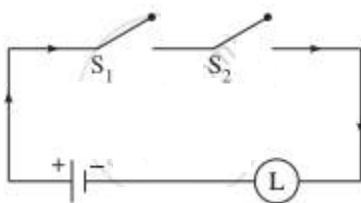
$$\equiv [(p \wedge q) \vee \sim (r \wedge s \wedge t)] \wedge [(p \wedge q) \vee (r \wedge s \wedge t)] \dots\dots\dots (\text{By De Morgan's Law})$$

$$\equiv (p \wedge q) \vee [\sim (r \wedge s \wedge t) \wedge (r \wedge s \wedge t)] \dots\dots\dots (\text{By Distributive Law})$$

$$\equiv (p \wedge q) \vee F \dots\dots\dots (\text{By Complement Law})$$

$$\equiv p \wedge q \dots\dots\dots (\text{By Identity Law})$$

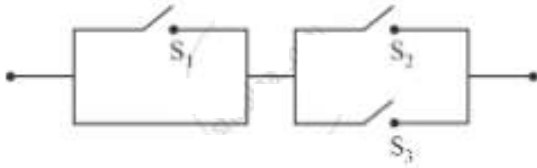
Hence, the alternative simplified circuit is:



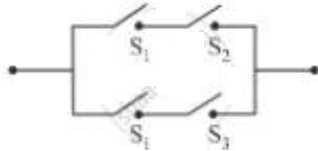
Miscellaneous Exercise 1 | Q 14.1 | Page 35

Check whether the following switching circuits are logically equivalent - Justify.

(i)



(ii)



Solution: Let p : the switch S_1 is closed

q : the switch S_2 is closed

r : the switch S_3 is closed

The symbolic form of the given switching circuits is $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$ respectively.

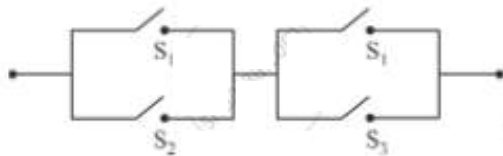
By Distributive Law, $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

Hence, the given switching circuits are logically equivalent.

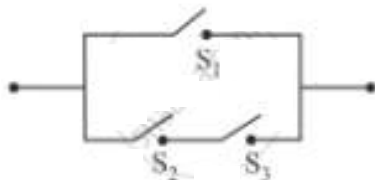
Miscellaneous Exercise 1 | Q 14.2 | Page 35

Check whether the following switching circuits are logically equivalent - Justify.

(i)



(ii)



Solution: Let p : the switch S_1 is closed

q : the switch S_2 is closed

r : the switch S_3 is closed

The symbolic form of the given switching circuits are

$(p \vee q) \wedge (p \vee r)$ and $p \vee (q \wedge r)$

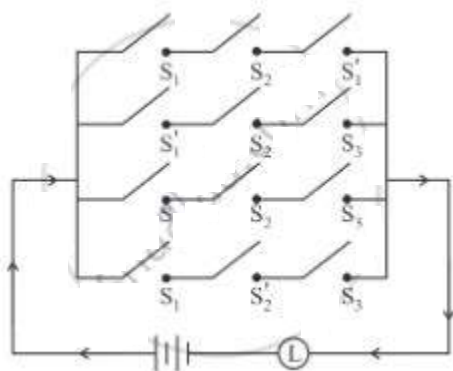
By Distributive Law,

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

Hence, the given switching circuits are logically equivalent.

Miscellaneous Exercise 1 | Q 15 | Page 35

Give alternative arrangement of the switching following circuit, has minimum switches.



Solution: Let p: the switch S_1 is closed

q: the switch S_2 is closed

r: the switch S_3 is closed

$\sim p$: the switch S_1' is closed or the switch S_1 is open.

$\sim q$: the switch S_2' is closed or the switch S_2 is open.

Then the symbolic form of the given circuit is:

$$(p \wedge q \wedge \sim p) \vee (\sim p \wedge q \wedge r) \vee (p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r)$$

Using the laws of logic, we have,

$$(p \wedge q \wedge \sim p) \vee (\sim p \wedge q \wedge r) \vee (p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r)$$

$$\equiv (p \wedge \sim p \wedge q) \vee (\sim p \wedge q \wedge r) \vee (p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r) \dots\dots\dots(\text{By Commutative Law})$$

$$\equiv (F \wedge q) \vee (\sim p \wedge q \wedge r) \vee (p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r) \dots\dots\dots(\text{By Complement Law})$$

$$\equiv F \vee (\sim p \wedge q \wedge r) \vee (p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r) \dots\dots\dots(\text{By Identity Law})$$

$$\equiv (\sim p \wedge q \wedge r) \vee (p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r) \dots\dots\dots(\text{By Identity Law})$$

$$\equiv [(\sim p \vee p) \wedge (q \wedge r)] \vee (p \wedge \sim q \wedge r) \dots\dots\dots(\text{By Distributive Law})$$

$$\equiv [T \wedge (q \wedge r)] \vee (p \wedge \sim q \wedge r) \dots\dots\dots(\text{By Complement Law})$$

$$\equiv (q \wedge r) \vee (p \wedge \sim q \wedge r) \dots\dots\dots(\text{By Identity Law})$$

$$\equiv [q \vee (p \wedge \sim q)] \wedge r \dots\dots\dots(\text{By Distributive Law})$$

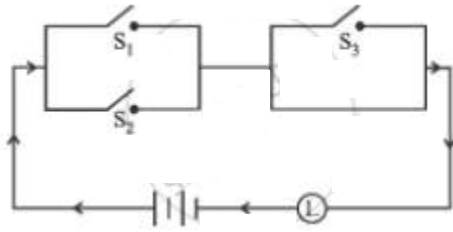
$$\equiv [(q \vee p) \wedge (q \vee \sim q)] \wedge r \dots\dots\dots(\text{By Distributive Law})$$

$$\equiv (q \vee p) \wedge T \wedge r \dots\dots\dots(\text{By Complement Law})$$

$$\equiv (q \vee p) \wedge r \dots\dots\dots(\text{By Identity Law})$$

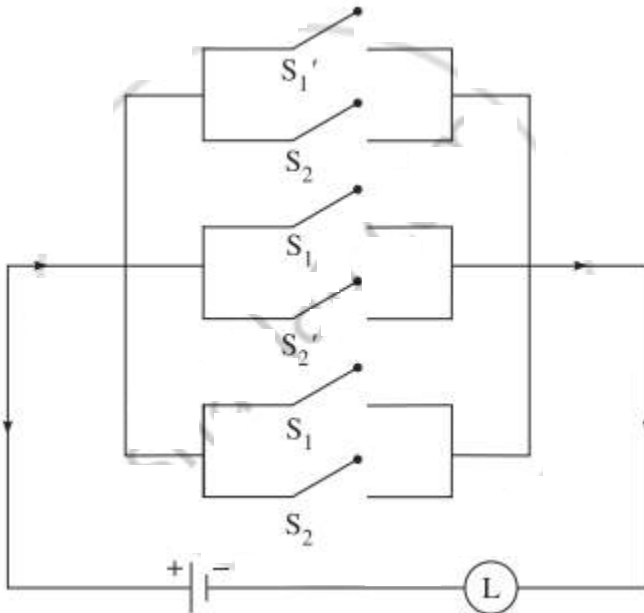
$\equiv (p \vee q) \wedge r$ (By Commutative Law)

\therefore the alternative arrangement of the new circuit with minimum switches is:



Miscellaneous Exercise 1 | Q 16 | Page 35

Simplify the following so that the new circuit.



Solution:

Let p: the switch S_1 is closed

q: the switch S_2 is closed

$\sim p$: the switch S_1' is closed or the switch S_1 is open

$\sim q$: the switch S_2' is closed or the switch S_2 is open.

Then the symbolic form of the given switching circuit is:

$$(\sim p \vee q) \vee (p \vee \sim q) \vee (p \vee q)$$

Using the laws of logic, we have,

$$(\sim p \vee q) \vee (p \vee \sim q) \vee (p \vee q)$$

$$\equiv (\sim p \vee q \vee p \vee q) \vee (p \vee q)$$

$$\equiv [(\sim p \vee p) \vee (q \vee q)] \vee (p \vee q) \dots\dots(\text{By Commutative Law})$$

$$\equiv (T \vee T) \vee (p \vee q) \dots\dots\dots(\text{By Complement Law})$$

$$\equiv T \vee (p \vee q) \dots\dots\dots(\text{By Identity Law})$$

$$\equiv T \dots\dots\dots(\text{By Identity Law})$$

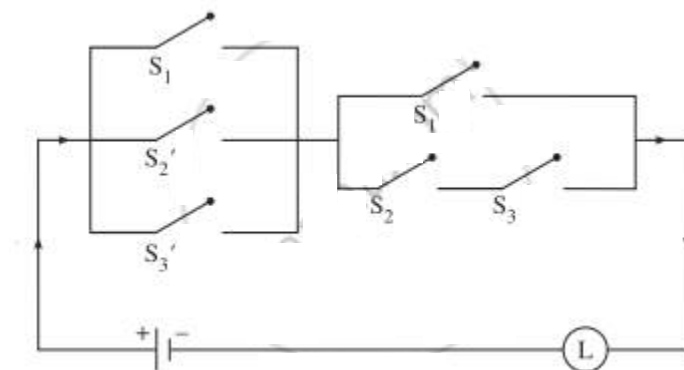
\therefore the current always flows whether the switches are open or closed. So, it is not necessary to use any switch in the circuit.

\therefore the simplified form of the given circuit is:



Miscellaneous Exercise 1 | Q 17 | Page 35

Represent the following switching circuit in symbolic form and construct its switching table. Write your conclusion from the switching table.



Solution:

Let p: the switch S_1 is closed

q: the switch S_2 is closed

r: the switch S_3 is closed

$\sim q$: the switch S_2' is closed or the switch S_2 is open

$\sim r$: the switch S_3' is closed or the switch S_3 is open

Then the symbolic form of the given circuit is:

$$[p \vee (\sim q) \vee (\sim r)] \wedge [p \vee (q \wedge r)]$$

Switching Table

p	q	r	$\sim q$	$\sim r$	$p \vee (\sim q) \vee (\sim r)$	$q \wedge r$	$p \vee (q \wedge r)$	Final column
					(I)		(II)	$(I) \wedge (II)$
1	1	1	0	0	1	1	1	1
1	1	0	0	1	1	0	1	1
1	0	1	1	0	1	0	1	1
1	0	0	1	1	1	0	1	1
0	1	1	0	0	0	1	1	0
0	1	0	0	1	1	0	0	0
0	0	1	1	0	1	0	0	0
0	0	0	1	1	1	0	0	0

From the table, the 'final column' and the column of p are identical. Hence, the given circuit is equivalent to the simple circuit with only one switch S_1 .

\therefore the simplified form of the given circuit is:

